Order of Operations

Now that you have practiced <u>operations</u> (+, −, •, and ÷) with Rational numbers, it's important to review the steps when a problem has more than one operation.

Step 1: Perform any operations that are in parentheses.

Step 2: Work with exponents, depending if the exponents are inside or outside the parentheses.

Step 3: Multiply or divide (or divide or multiply), which ever is first from left to right.

Step 4: Add or subtract (or subtract or add), which ever is first from left to right.

Example 1: Simplify $5 - 18 \div 3 \cdot 2$

$$5 - 6 \cdot 2$$ From left to right, divide first.

$$5 - 12$$ Multiply next.

$$-7$$ Now subtract from left to right.

Work down showing each step. You should end up with your work looking like a funnel.

Example 2: $-3 - 5\frac{1}{2} \cdot -\frac{1}{11} + 2$

$-3 + -\frac{11}{2} \cdot -\frac{1}{11} + 2$ From left to right, multiply first.

$-3 + \frac{1}{2} + 2$ Notice two changes were made because of the subtraction sign.

$-\frac{1}{2}$ Add from left to right.

Example 3: $-8 - (2 + 5)^2 + 9$

$-8 - 7^2 + 9$ Do parentheses first.

                      Then the exponent.

$-8 - 49 + 9$ Go left to right.
                      $-8 - 49$ is the same as
$-57 + 9$ $-8 + -49$

-48

Example 4: $90 \div -5 \cdot (3^2 + 9)$

                      Do the exponent first since
                      it's inside the parentheses.

$90 \div -5 \cdot (9 + 9)$ Now, do the parentheses.

$90 \div -5 \cdot 18$ Go left to right.

$-18 \cdot 18$

-324

Example 5: $\dfrac{6.59 + 11.02}{-1 + 4}$ This division line acts like
                      parenthesis for the numerator
                      and parentheses for the
                      denominator.

$\dfrac{17.61}{3}$ Simplify the numerator first.
                      Simplify the denominator.

5.87 Divide.

Example 3: What percent of 20 is 5? (Case 3)

What percent means that you don't know the rate. Write "what percent" as $\dfrac{n}{100}$. Remember that 20 is after the word "of" so it's the whole or base. Here's the proportion.

$$\frac{n}{100} = \frac{5}{20}$$

$20n = 500$ Now divide by 20.

$\quad n = 25$ The answer is 25%

> Why is 25% of 20 equal to 5? It's because $\dfrac{1}{4}$ of 20 = 5.

Example 4: What percent of 8 is 7? (Case 3)

$$\frac{n}{100} = \frac{7}{8}$$

$\quad 8n = 700$ Now divide by 8.

$\quad n = 87.5$ The answer is 87.5%

> Did you recognize that $\dfrac{7}{8}$ is 87.5%?

Case 1 can also be solved by using a proportion although it's not as fast as multiplying the rate times the base.

Example 5: Find 18% of 360.

18% is $\dfrac{18}{100}$. The base or whole is 360 since it's after the word "of," so it goes in the denominator. Let n be the unknown. You're finding the percentage.

$$\frac{18}{100} = \frac{n}{360}$$

$100n = 6,480$

$\quad\quad n = 64.8$

Practice

Simplify the following problems. If you end up with a negative exponent, write your answer two ways (see Example 2). Assume none of the variables in the denominator are zero.

1. $\dfrac{-10a}{-5a}$ _____

2. $\dfrac{40c^4}{2c^9}$ _____

3. $\dfrac{-26a^6}{13a^7}$ _____

4. $\dfrac{2a^6}{200a^8}$ _____

5. $\dfrac{-90x^8}{-90x^9}$ _____

6. $\dfrac{18m^6n}{-9m^2n^8}$ _____

7. $\dfrac{-88mn^8}{-11m^9n}$ _____

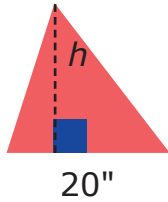8. $\dfrac{-50w}{-75w^3}$ _____

9. $\dfrac{-50}{-25w}$ _____

10. $\dfrac{-34x}{-68x^9}$ _____

11. $\dfrac{80}{-40x^9}$ _____

12. $\dfrac{3^3}{3^4}$ _____

Using Algebra to Solve Geometry Problems

Example 1: A triangle has base of 20 inches and an area of 60 square inches. Find the height of the triangle.

Draw a picture (don't worry if it's not to scale).
Write the formula:

$\frac{1}{2}bh = A$

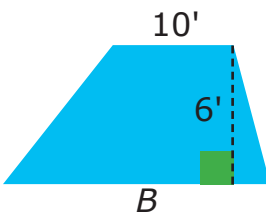$\frac{1}{2} \cdot 20 \cdot h = 60$ Substitute.

$10h = 60$ Divide by 10 on each side.

$h = \boxed{6}$

Answer: The height is 6".

Example 2: A trapezoid has a height of 6' and a small base of 10'. Its area is 72 square feet. Find the larger base.

$\frac{1}{2}(b + B) \cdot h = A$

$\frac{1}{2}(10 + B) \cdot 6 = 72$ Substitute and distribute.

$(5 + \frac{B}{2}) \cdot 6 = 72$ Divide by 6 on each side, or you could distribute again.

$5 + \frac{B}{2} = 12$ Subtract 5 from each side.

$\frac{B}{2} = 7,$ Multiply each side by 2.

So, $B = 14$

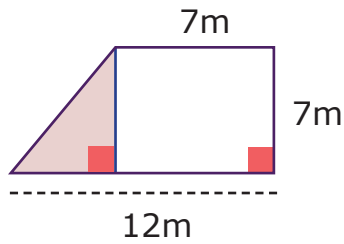Answer: The larger base is 14'.

Practice

Solve the following problems using algebra. Start by writing down the area formula.

1. A triangle has an area of 600 square meters and a base of 300 meters. Find its height. _____
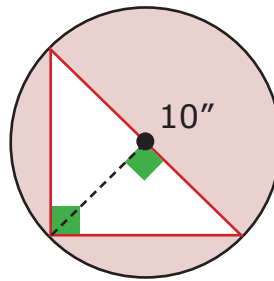
Find the shaded area.

7m

29.

7m

12m

$A =$ _____

30.

10"
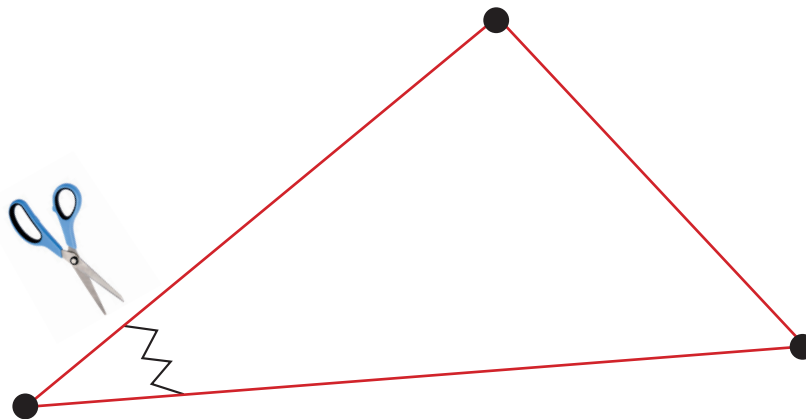
$A =$ _____

A right triangle is inscribed inside a circle with diameter of 10". Use 3.14 for PI.

31. Finding the Sum of the Measures of the Angles in a Triangle

    a. Take a sheet of paper and trace the following triangle or make your own triangle with a ruler. Make sure you also darken the vertices as below.

    b. Cut out the triangle and cut out the angles, making sure you know where the vertices are located after you cut the angles.

    c. Place all the angles together on the line below. The angles should line up on the line (make sure you put the vertices together) demonstrating that the angle measures of a triangle have a sum of 180º.

11.

x	−1	0	1	2	3	4
f(x)	$\frac{1}{3}$	1	3	9	27	81

Type: _____   Function: _____

12.

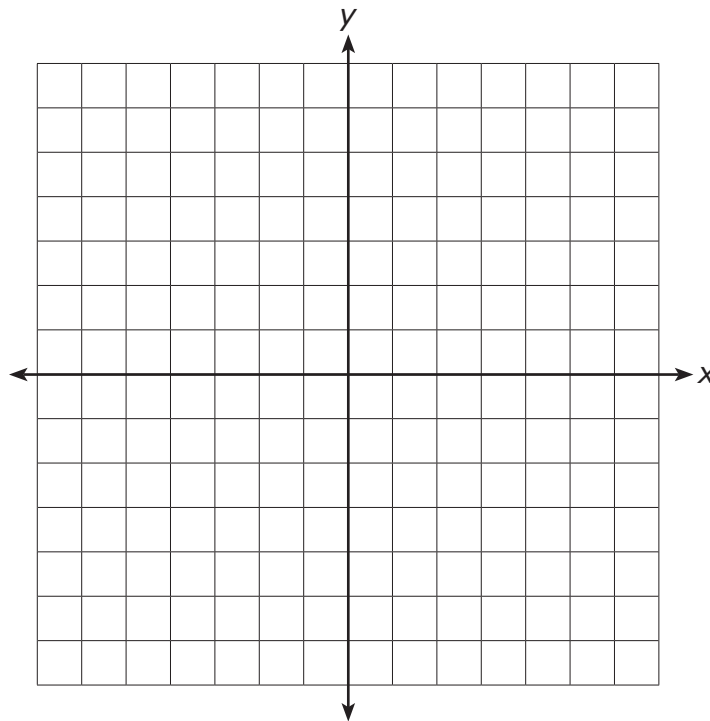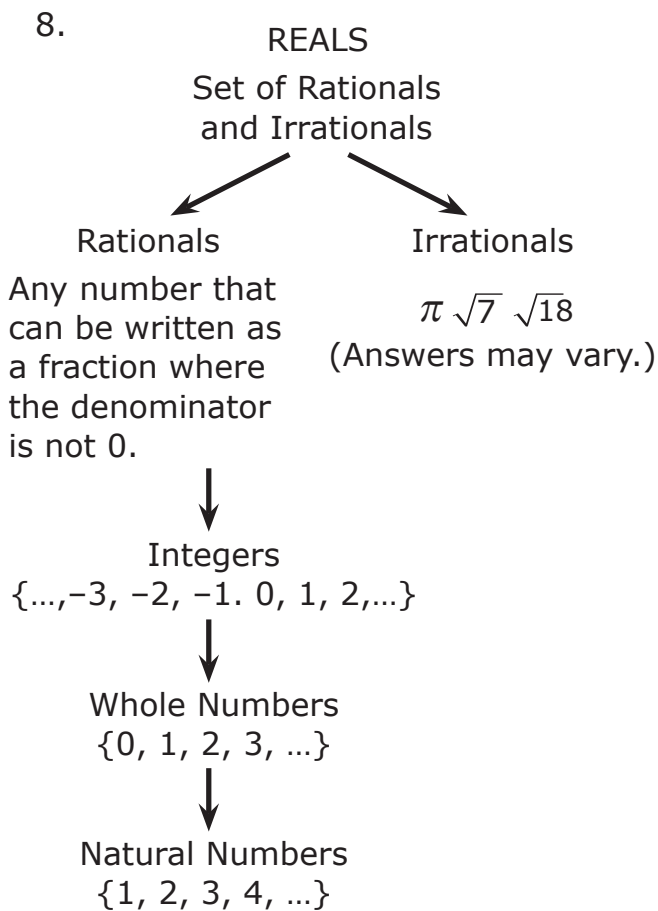| x	−3	−2	−1	0	1	2
f(x)	9	6	3	0	−3	−6

Type: _____   Function: _____

13. Fill in the chart, then graph this absolute value function: $y = |x| - 4$.

x	f(x)
−4	
−2	
0	
2	
4	

14. Compare the graph for problem #13 with the graph of $y = |x|$. What transformation would move one function into the other?

_____

_____

8.

REALS
Set of Rationals
and Irrationals

Rationals

Any number that
can be written as
a fraction where
the denominator
is not 0.

Irrationals

π $\sqrt{7}$ $\sqrt{18}$
(Answers may vary.)

Integers
{…,−3, −2, −1. 0, 1, 2,…}

Whole Numbers
{0, 1, 2, 3, …}

Natural Numbers
{1, 2, 3, 4, …}

Pages 13-14 Chapter 1 Review

1. e

2. c

3. a

4. b

5. d

6. (Answers may vary.)
    π is irrational, so it is real.
    −3 is an integer, so it it real.
    6.5 is a rational, so it is real.

7. The numbers are all perfect
    squares. (13 • 13 = 169,
    15 • 15 = 225, and 17 • 17 = 289)

8. a. 5 • 5 • 5 or 5^3

    b. 2 • 2 • 2 • 3 • 3 or 2^3 • 3^2

    c. 2 • 2 • 23 or 2^2 • 23

    d. 2 • 2 • 2 • 2 • 2 • 2 • 3 • 5 or
       2^6 • 3 • 5

9. a. F; b. T; c. F; d. F; e. F;
    f. T; g. T

10. a. No. 75 is divisible by 5 and 25.

    b. No. 91 is divisible by 7 and 13.

    c. Yes. 97 is prime. Its only
       factors are 1 and 97.

    d. No. 111 is divisible by 3 and 37.

    e. (Answers may vary.)
       No. 120 is divisible by 2 and 10.

11. a. RAT; b. RAT; c. RAT; d. RAT;
    e. RAT; f. IRRAT

12. (Answers may vary.) The number
    36 is divisible by 2 and 3 and it is
    therefore divisible by 6.

Pages 61–62

Recipe	Eggs	Flour	Brown Sugar	Unsalted Butter
1	4	$4\frac{1}{2}$c	$1\frac{1}{2}$c	1c
2	$2\frac{2}{3}$	3c	1 cup	$\frac{2}{3}$c

1. 4 tsp vanilla extract (The recipe was multiplied by 4)

2. 3 tsp baking soda ($\frac{3}{4}$ • 4)

3. $\frac{2}{3}$ c unsalted butter. The recipe was changed by multiplying by $\frac{4}{3}$, so

    $\frac{4}{3} • \frac{1}{2} = \frac{2}{3}$

4. a. 42 mi/gal
    b. 8 gallons
    c. 499.8 miles

5. 216 words; $\frac{90}{75} = \frac{n}{180}$

6. a. 557 lbs   Eva   Dad

        $\frac{100\ Earth}{236\ Jupiter} = \frac{236\ Earth}{n}$

    b. 90 lbs   Eva   Dad

        $\frac{100\ Earth}{38\ Mars} = \frac{236\ Earth}{n}$

    c. 90 lbs   Eva   Sister

        $\frac{100\ Earth}{236\ Jupiter} = \frac{38\ Earth}{n}$

7. 144 apples $\frac{4.50}{12} = \frac{54}{n}$

8. 500 pages $\frac{24}{20} = \frac{600}{n}$

Pages 64-66

1. 25” X 37.5”; divide by 8

2. 628.6 miles; $\frac{1.4}{400} = \frac{2.2}{n}$

3. 1 cm = 3 meters; $\frac{72}{24} = \frac{3}{1}$

4. Drawings will vary.

5. 90 inches $\frac{10cm}{60in} = \frac{15cm}{n}$;

    5,400 sq in. (area) or 37.5 sq ft

6. 2.5 inches $\frac{\frac{1}{4}}{20mi} = \frac{n}{200mi}$

7. 2 m X 1.9 m X 1.68 m
    (200 cm X 190 cm X 168 cm);
    Volume: 6.384 cubic meters

8. 48.75 inches or 4’ 9” $\frac{1.5”}{n} = \frac{2”}{65”}$

9. a. 1:3

    b. Figure A: 14”, Figure B: 42”

    c. 1:3 (14:42)

    d. Figure A: 10 sq in.,
        Figure B: 90 sq in.

    e. 1:9 (10:90)

Page 69

1. $\frac{1}{4}$

2. $\frac{2}{1}$ or 2

3. $\frac{1}{40}$

4. 12.5%

4. $\dfrac{1}{100a^2}$ or $\dfrac{a^{-2}}{100}$

5. $\dfrac{1}{x}$ or x^{-1}

6. $\dfrac{-2m^4}{n^7}$ or $-2m^4n^{-7}$

7. $\dfrac{8n^7}{m^8}$ or $8m^{-8}n^7$

8. $\dfrac{2}{3w^2}$ or $\dfrac{2w^{-2}}{3}$

9. $\dfrac{2}{w}$ or $2w^{-1}$

10. $\dfrac{1}{2x^8}$ or $\dfrac{1}{2}x^{-8}$ or $.5x^{-8}$

11. $\dfrac{-2}{x^9}$ or $-2x^{-9}$

12. $\dfrac{1}{3}$ or 3^{-1}

Page 111

1. 5.88; 8

2. –9

3. 10^9

4. –6

5. 3×10^{-6}

6. 1×10^{10}

7. 3.24×10^8

8. 1.3×10^{-7}

9. 1,387,000,000

10. .000000000025

11. 186,000

12. 238,900

Pages 112–113

1. 7.5×10^{12}

2. 9×10^{11}

3. 2×10^3

4. 1.408×10^{14}

5. 5×10^{-1}

6. 9×10^9

7. 1.95×10^{13}

8. 2×10^4

9. 6×10^{-7}

10. 2.5×10^{10}

Pages 114–116 Chapter 6 Review

1. d

2. c

3. a

4. c

5. c

6. d

7. a

8. b

Pages 272–275 Chapter 12 Review

1. II

2. IV

3. III

4. y-axis

5. x-axis

6. I

7.
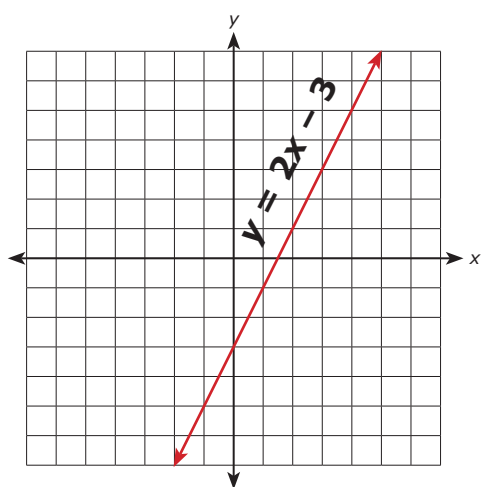
x	y
−2	−7
−1	−5
0	−3
1	−1
2	1
3	3

8.
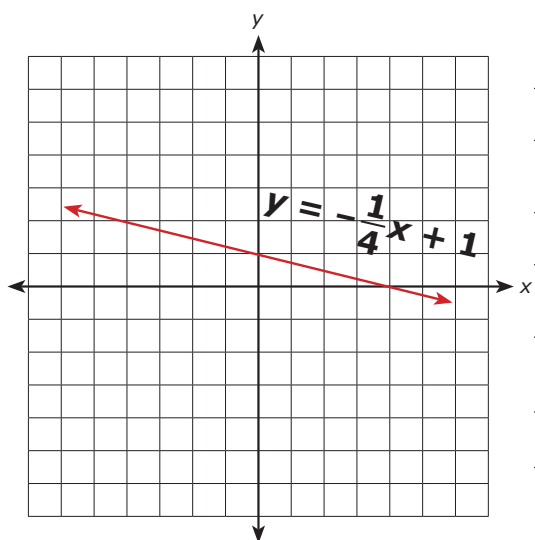
x	y
−4	2
−2	$1\frac{1}{2}$
0	1
2	$\frac{1}{2}$
3	$\frac{1}{4}$
4	0

9. $y = 3x - 6$

10. $y = -\frac{1}{2}x + \frac{1}{2}$
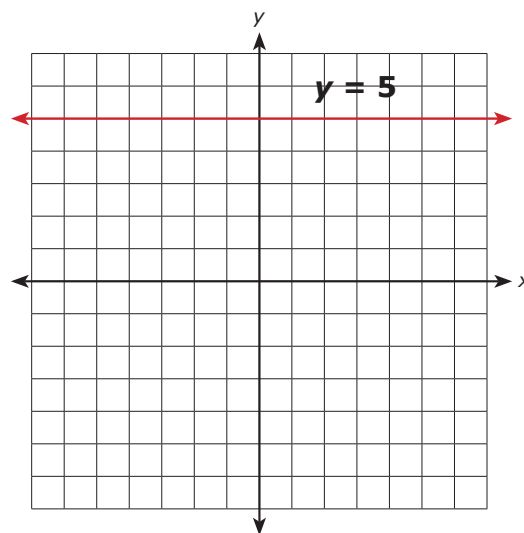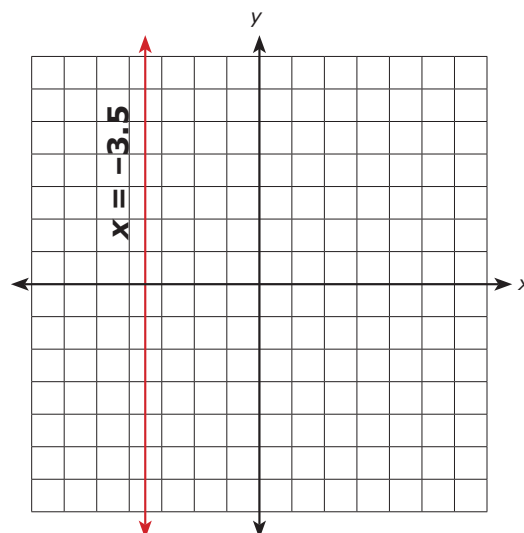
11. $y = -2x - 6$

12. $y = 4x - 6$

13. $y = -2x + 3$

14. $y = 2x + 8$

15.

16.

2. (−3, 2)

3. (3, 2); (−3, 2)

4. They both have the same coordinates. Reflecting about the origin is the same as reflecting about one axis and then about the other axis.

5. a. (−3, 4)

    b. (5, −6)

    c. (2, 5)

    d. (0, 10)

Page 282

1. Answers will vary.

2. a. (3, −2); b. (5, 0);
    c. (−5, 3); d. (30, 20)

3. a. (3, 1); b. (3, 4)
    c. (5, 4); d. (5, 1)

4. Yes, the corresponding sides remain congruent and the corresponding angles are also congruent.
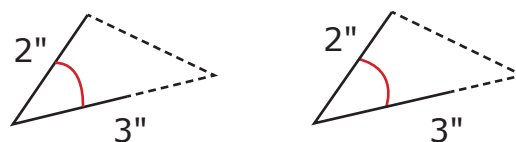
Pages 284−285

1. The x and y values changed signs.

2. a. (−1, −2); b. (−6, −6);
    c. (−5, −1)

3.
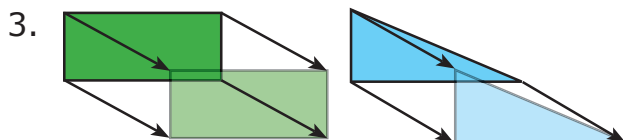
4. A' (−1, 3); B' (2, 0); C' (−4, −4)

5. a. Q' (-1, 6); R' (−2, 2); S' (−6, 1)

    b. Q' (−1, −6); R' (−2, −2); S' (−6, −1)

    c. Q' (6, −1); R' (2, −2); S' (1, −6)

Pages 287−289

1. Yes, fixing the angle between the corresponding congruent sides will guarantee congruency.

2. Both triangles are congruent.

When two sides are congruent to the two corresponding sides of another triangle and the angle between those two sides is also congruent in both triangles then the two triangles are congruent.

3. Yes, SSS

4. Yes, SAS

5. cpctc; Corresponding parts of congruent triangles are congruent.

6. No, the triangle on the left is SAS, and the one on the right is SSA.

7. Triangles sides are extended to meet at one point.

    b. Yes, if you extend the sides you create two congruent triangles. Both triangles have two corresponding angles congruent

4. Graph 2b is 3 units down and a translation of -3 ($T_{0,-3}$) compared the the basic absolute value function.

5. It will be down 20 units from the basic absolute value function.

6.

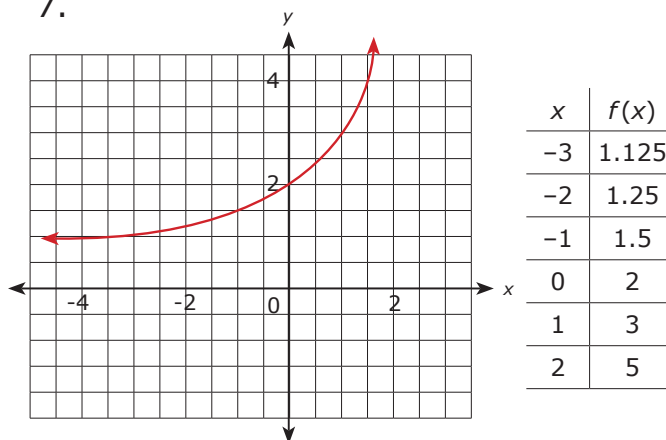| x	f(x)
-2	7
0	3
2	-1
4	-5
6	-1
8	3
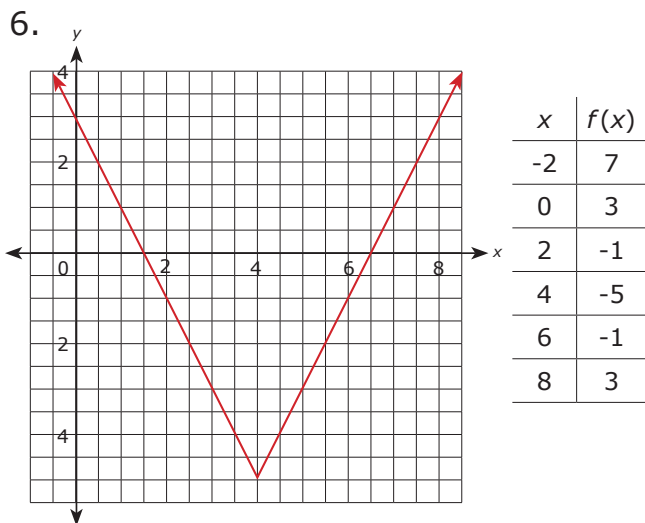
Pages 319–322 Chapter 14 Review

1. c; The input −4 has two different outputs.

2. a. {−3, −2, −1, 0, 1, 2, 3, 4)

   b. {9, 4, 1, 0, 1, 4, 9, 16}

   c. $y = x^2$ or $f(x) = x^2$

   d. Yes, any real number can be substituted for x.

   e. The range will consist of positive real numbers greater than or equal to 0.

3. It does not meet the vertical test. Example: (0, 3) and (0, −3).

4. d

5. a

6.

x	f(x)
−3	5
−2	0
−1	−3
0	−4
1	−3
2	0
3	5
4	12

7.

x	f(x)
−3	1.125
−2	1.25
−1	1.5
0	2
1	3
2	5
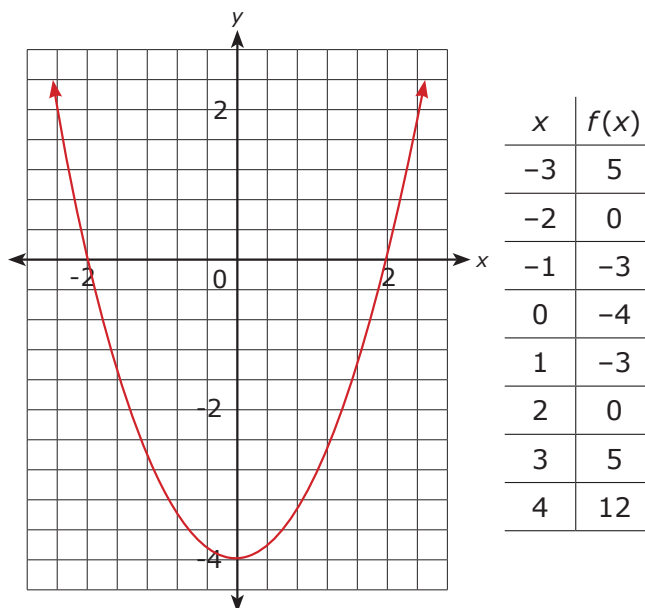
8. Ordered pairs from the table:

   (−2, 9); (−1, 3); (0, 1); (1, $\frac{1}{3}$);
   (2, $\frac{1}{9}$); (3, $\frac{1}{27}$)

9. Linear, $y = 5x + 1$

10. Quadratic, $y = 2x^2$

11. Exponential, $y = 3^x$

12. Linear, $y = -3x$

Pages 333–334

1. a. 12   (3 • 2 • 2)

    b. 6   (3 • 2)

    c. Answers will vary.

2. 24   (4 • 3 • 2 • 1)

3. 120 (5! = 5 • 4 • 3 • 2 • 1)

4. 3,125 (5^5)

5. 60   (5 • 4 • 3)

6. 24 (**2** • 4 • 3) Only two choices to get a number greater than 700.

7. 36   (3 • 4 • **3**) Only three choices for the last digit to be odd.

8. 12   (3 • 4 • **1**) Only one choice for the last digit to create a number divisible by 5.

9. 8   (4 • **2**) Only two choices to create an even number.

10. 16, 18, 56, 58, 68, 76, 78, 86

Pages 335–336

1. Combination 8*C*4 = 70 teams.

2. Permutation 8*P*3 = 336 outcomes.

3. Combination 5*C*2 = 10 logos.

4. 120 (6*P*3 = 6 • 5 • 4)

5. a. 336 (8*P*3)

    b. $\frac{6}{336} = \frac{1}{112}$; Three female positions: 3 • 2 • 1 = 6

6. 6 (4 • 3 ÷ 2) *BE, EB, ES, SE, BS, SB*

Page 337

1. Answers will vary.

2. The probability will approach $\frac{1}{2}$ or 50% the more times you do the experiment.

Pages 339–340

1. The median. The $100,000 salary is much higher when compared to the other salaries, so the mean misrepresented what she's most likely to earn.

2. Mode: none
    Range: $29,500-$100,000
    Range: $70,500

3. Mean: 3; Median: 2; Mode: 2; Range: −1 to 12 or 13 degrees.

4. The mean is more representative of her overall grade.

5. The mode since most students said No.

6. Mean: .569 (In order: 0.03, 0.09, 0.222, 0.88, 0.95, 1.24)

    Median: .551 (.222 + .88) ÷ 2

7. Answers will vary.

8. Answers will vary.

9. 86
    Solve: (88 + 89 + 92 + x + 95) ÷ 5 = 90)