Table of Contents

Using This Book

This all-in-one set of lessons enables both the student and teacher/parent to cooperately work together through fun, yet challenging activities. A high-level mastery of fractions and percent is inevitable with this step-by-step, hands-on approach! And by rounding it off with an introduction to decimals, students will understand the interrelatedness between fractions, percent, and decimals and be at ease in moving between them as needed in solving real-life problems.

To ensure a smooth start, it is most important that the student attempts the initial three pages without adult help. If they struggle with more than a few problems, the teacher should consider having them review the first book in this series, *Understanding Fractions*.

As new topics are introduced, jointly read over the pages while using the provided cut-out fraction bars and/or hundreds grid – see below. This will boost students understanding and motivation to apply themselves during the follow-up practice. Spot-checking those completed worksheets by asking students to explain (justify) their reasoning on a few problems is mathematically valuable as it deepens their understanding and enhances their ability to mentally manipulate fractions.

Multiplication Table, Decimal 100's Grid, and Fraction Bars

There is a multiplication table cut-out on page 65. Be sure to review how to use it and importantly become a 'warm demander' encouraging they memorize these facts, as it is the single most vital step in developing confidence in their math abilities going forward. The decimal 100's grid cut-out is also on page 65. This is useful when working through percent and decimals activities.

The fraction bars cut-outs on page 67 are used as support and free exploration for students to "see" how fractions work. These can provide a helpful aid when topics are introduced (follow along with the examples in the lesson) as well as during practice exercises.

- To increase durability, glue or paste the page to heavier stock, such as a file folder or thin cardboard, prior to cutting out
- Fastening an envelope to the inside back cover will allow storage of the cut-outs.

A free PDF (www.CriticalThinking.com/tablefractions) of these cut-outs is also available to download and print.

Equivalent Fractions

Remember, multiplying or dividing fractions by 1 does not change their value, only their appearance. Also, 2 out of 2 parts or $\frac{2}{2}$ means one whole. Similarly, $\frac{3}{3}$ . . . $\frac{100}{100}$ . . . all equal 1.

Changing a fraction into an equivalent fraction, whether larger or reduced to lowest terms, is quicker and more fun when you've memorized the multiplication table. Until then, use the cut-out table on page 59 to help you.

How to Use the Multiplication Table*

To solve $\frac{3}{4} = \frac{\square}{28}$, what form of 1 is needed to multiply $\frac{3}{4}$ by? Well, what times 4 = 28? Look down the 4's column until you see 28. Then look across that row to find the outside number is 7. That tells us 7 x 4 = 28 and to use $\frac{7}{7}$ as the multiplier. So, $\frac{7}{7} \times \frac{3}{4} = \frac{7 \times 3}{4 \times 4} = \frac{\square}{28}$. So what is 7 x 3? Move one finger down the 7's column and one finger across the 3's row. They meet at 21. So, 7 x 3 = 21 and therefore $\frac{3}{4} = \frac{21}{28}$.

x	1	2	3	4	5	6	7	8
1	1	2	3	4	5	6	7	8
2	2	4	6	8	10	12	14	16
3	3	6	9	12	15	18	21	
4	4	8	12	16	20	24	28	
5	5	10	15	20				
6	6	12	18	24				
7	7	14	21	28				

* Students should explore how the multiplication table is repeated addition or skip counting if they are not clear, e.g., 3 x 7 is the same as 7 + 7+ 7, and the same as seven 3's.

Reciprocals

When a fraction is flipped upside down, the denominator becomes the numerator and the numerator becomes the denominator. This is called the reciprocal (re-sip-ro-cull) or inverse. To find the reciprocal of $\frac{3}{4}$, invert the fraction to $\frac{4}{3}$.

Multiplying a fraction by its reciprocal always makes the product of 1.

For example: $\frac{2}{3} \times \frac{3}{2} = \frac{6}{6} = 1$; $\frac{9}{10} \times \frac{10}{9} = \frac{90}{90} = 1$. Remember, a whole number is assumed to be a fraction with a denominator of 1 ($5 = \frac{5}{1}$). So its inverse is $\frac{1}{5}$ in this case.

Mixed numbers need to be changed to improper fractions to find their reciprocals. For example, $3\frac{1}{2} = \frac{7}{2}$ so $\frac{2}{7}$ is the inverse.

Shortcut: To change a mixed number to an improper fraction, multiply the whole number by the denominator, then add the numerator. Put this new number over the original denominator.
Examples: $2\frac{1}{3} = (3 \times 2) + 1 = \frac{7}{3}$. $4\frac{3}{5} = (5 \times 4) + 3 = \frac{23}{5}$.

Write the reciprocal for these mixed numbers in the blank.

$\frac{2}{5}$ 1. $2\frac{1}{2} = \frac{5}{2}$	____ 2. $3\frac{3}{4} =$	____ 3. $6\frac{1}{5} =$
____ 4. $7\frac{1}{8} =$	____ 5. $3\frac{5}{9} =$	____ 6. $8\frac{1}{8} =$
____ 7. $9\frac{1}{5} =$	____ 8. $5\frac{3}{8} =$	____ 9. $7\frac{3}{4}$
____ 10. $6\frac{5}{6} =$	____ 11. $7\frac{4}{8} =$	____ 12. $9\frac{1}{7} =$
____ 13. $9\frac{1}{5} =$	____ 14. $4\frac{3}{8} =$	____ 15. $10\frac{4}{5}$
____ 16. $50\frac{1}{3} =$	____ 17. $9\frac{99}{100} =$	____ 18. ★ $99\frac{999}{10,000}$

Multiplying Fractions Shortcut*

Take the problem $\frac{3}{4} \times \frac{4}{5}$. Notice that the numerator and denominator are both multiplied by 4. This means that you will also divide by 4 when reducing the new fraction to its simplest form. Let's see: $\frac{3 \times 4}{4 \times 5} = \frac{12}{20} \div \frac{4}{4} = \frac{3}{5}$

A shortcut is to "cancel" out the 4's before doing the multiplying and dividing.

A number is canceled with a single strike-through. Since 4 ÷ 4 = 1, a small 1 is put next to both canceled numbers.

$$\frac{3}{\underset{1}{\cancel{4}}} \times \frac{\overset{1}{\cancel{4}}}{5} = \frac{3 \times 1}{1 \times 5} = \frac{3}{5}$$

You can cross-cancel a numerator anytime it will evenly divide into a denominator, or the other way around. Explain these examples step-by-step.

a. $\frac{5}{6} \times \frac{7}{10} = \frac{\overset{1}{\cancel{5}}}{6} \times \frac{7}{\underset{2}{\cancel{10}}} = \frac{1 \times 7}{6 \times 2} = \frac{7}{12}$

b. $\frac{6}{8} \times \frac{4}{6} = \frac{\overset{1}{\cancel{6}}}{\underset{2}{\cancel{8}}} \times \frac{\overset{1}{\cancel{4}}}{\underset{1}{\cancel{6}}} = \frac{1 \times 1}{2 \times 1} = \frac{1}{2}$

c. $1\frac{3}{5} \times 7\frac{1}{2} = \frac{\overset{4}{\cancel{8}}}{\underset{1}{\cancel{5}}} \times \frac{\overset{3}{\cancel{15}}}{\underset{1}{\cancel{2}}} = \frac{4 \times 3}{1 \times 1} = 12$

d. $1\frac{2}{4} \times \frac{4}{5} \times \frac{2}{3} = \frac{\overset{2}{\cancel{6}}}{\underset{1}{\cancel{4}}} \times \frac{\overset{1}{\cancel{4}}}{5} \times \frac{2}{\underset{1}{\cancel{3}}} = \frac{2 \times 1 \times 2}{1 \times 5 \times 1} = \frac{4}{5}$

Multiply the following fractions using cross-cancelling when possible.

1. $\frac{4}{7} \times \frac{1}{4}$
2. $\frac{5}{8} \times \frac{2}{5}$
3. $\frac{4}{6} \times \frac{6}{16}$
4. $\frac{3}{4} \times \frac{20}{30}$
5. $\frac{2}{3} \times \frac{12}{14}$
6. $\frac{2}{6} \times \frac{24}{30}$
7. $1\frac{1}{4} \times \frac{12}{20}$
8. $3\frac{1}{5} \times \frac{3}{8}$
9. $4\frac{1}{6} \times 3\frac{3}{5}$
10. $8\frac{3}{4} \times 1\frac{3}{5}$
11. ★ $1\frac{1}{3} \times \frac{6}{8} \times \frac{4}{5} \times 1\frac{1}{4}$

*See page 54 SET C for extra practice.

More Than 100%

Example: 120% of 10 = ? $\frac{120}{\cancel{100}_{10}} \times \frac{\cancel{10}^{1}}{1} = \frac{120}{10} = 12$

Check: 100% of 10 = 10; 20% of 10 = 2; so 120% = 10 + 2 = 12

You may find it easier to convert the percent to a fraction before starting.

Example: 250% of what number makes 10? 250% = $\frac{250}{100} = 2\frac{1}{2}$.

$$2\frac{1}{2} \quad \times \quad \square \quad = \quad 10$$

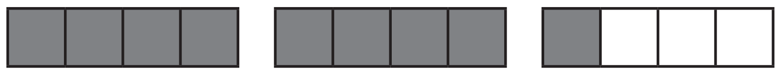
$10 \div 2\frac{1}{2} = \frac{10}{1} \times \frac{2}{5} = 4$. Check: Is $2\frac{1}{2} \times 4 = 10$? Yes.

Solve each problem. Show your work.

1. 110% of 50 =
   $1\frac{1}{10} \times 50$
2. 200% of 35
3. 175% of 20
4. 450% of 1
5. 140% of 5
6. $166\frac{2}{3}$% of 900
7. 250% of $\frac{3}{5}$
8. 325% of 300
9. A library loans 600 books daily. It had $133\frac{1}{3}$% increase in borrowed books. What is the new total?
10. A teacher assigns 20 math problems. A student does those plus 3 extra credit problems. What percentage of the assignment did the student do?
11. 225% of me is 90. What number am I?
12. What percent of 20 is 110?
13. What percent of $\frac{1}{4}$ is $\frac{1}{10}$?
14. ★ $1\frac{2}{3}$ is what percent of $\frac{1}{4}$?

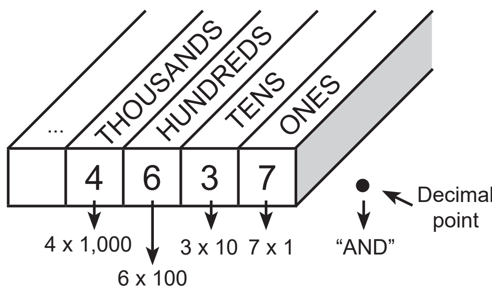
Fractions as Decimals

This amount of money can be described as $1\frac{1}{2}$ dollars, but it is sometimes more convenient and easier to understand when the fraction part is written using a decimal point as in $1.50. This is read as “one dollar and fifty-cents” but also as “one and fifty hundredths” ($1\frac{50}{100}$) dollars, since $\frac{1}{2} = \frac{50}{100}$. The decimal point is read as “and,” it separates the whole from the fraction.



$2\frac{1}{4} = 2\frac{25}{100} = 2.25$ (two and twenty-five hundredths).

To understand the math connection between fractions and decimals, below is a short review of base 10 or the decimal system. The place value system is based on ones, tens, and multiples of tens (10 x 10, 100 x 10, 1000 x 10 …)



Whole numbers, such as 4,637 are usually written without the decimal point since there is no fractional part to separate out. The decimal point is implied to be there, as in 4,637.0, but is unnecessary to write.

Expanded notation is a good way to understand the decimal system.

For example:

(a) 365

(3 x 100) + (6 x 10) + (5 x 1)

(b) 1,492

(1 x 1,000) + (4 x 100) + (9 x 10) + (2 x 1)

Just For Fun: Decimal comes from the Latin root word “Decem” meaning 10 or 10th. A decade is 10 years and a decimeter is 1/10 of a meter. In the old Roman calendar, December was the 10th month!