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About The Author

Terri Husted is a retired mathematics teacher who still works for the Ithaca City School district in Ithaca, NY, as an Educational Coach. She has written many math workbooks to help students. She taught mathematics in middle school and high school for over 33 years and still misses being in the classroom. In her free time she likes to write, knit, garden and dance. She has Math certification grades 7-12 and Reading K-12. Her specialty is Reading in the Content Area of Math and in Multicultural Math.

Special Thanks

Thank you to my grandson Paulo Acevedo who helped me understand why a favorite subject like math can become frustrating at times. He helped me by highlighting the concepts he found difficult. To Lisa Alexander, an amazing mathematics teacher at Ithaca High School, Ithaca, NY, and former colleague, who shared with me the common mistakes students make year after year. And to all my former students, who inspired me to become a better teacher the more I taught.

Introduction

Algebra is essentially the language of all advanced mathematic courses. It is generalized arithmetic, so the rules that apply when we are working with number and fraction operations—including the knowledge of factors and multiples—also apply when we are working with algebra.

As an experienced high school math teacher, I have watched countless students of all ability levels struggle with more advanced high school mathematics, simply because they lack a solid understanding of critical math concepts taught in earlier grades that are applied in algebra. For example, when students begin working with variables in Algebraic equations, they are expected to apply the knowledge and skills they learned years ago multiplying and dividing fractions, but many have forgotten or never thoroughly understood these earlier concepts.

Without this knowledge and skills, students often struggle or even fail in advanced mathematics courses. Imagine a good high school student who sees a problem like 3•x•y•4 and hesitates to write 12xy because he or she is unsure of the rules that govern multiplication. Or they don't understand how to add 2x to 1/4y to combine it into one entire fraction. Or why −62 is different than (−6)2. It is easy to see that not having a strong understanding of the foundational rules of algebra can stop even the smartest students from succeeding in advanced high school math courses.

The in-depth approach to math skills in this book focuses on the important algebraic concepts that are vital to success in advanced high school mathematics. It teaches the fundamental arithmetic principles that govern algebra, so that students can apply them as needed in advanced courses, and in life. It begins with the definition of a "term" and the facts of when you can cancel and when you cannot. It discusses the importance of factors and multiples and integer rules. It teaches addition, subtraction, multiplication, and division of fractions – not by doing the problems up and down like they are mostly done in elementary school, but by doing them sideways just like they are done later in Algebra and all higher math. It makes everything so much easier down the road!

The explanations are clear and followed with basic step by step procedures to show students how to master and apply those fraction concepts, negative number operations, exponent rules, work with square roots, and doing real life percent problems with charts and tables, just like those on the SAT's. I purposely repeat many of these concepts in different settings, so students will keep re-applying what they learn throughout the book. The language is friendly to put students at ease and increase their confidence while learning concepts that are often confusing.

This book reviews the necessary reading strategies needed to translate word problems, but it is not the primary focus of the book. My book Understanding Algebra I devotes large sections to many types of word problems and many useful strategies if deeper study of this topic is desired.

In the last two chapters of the book, I help students apply the algebra they have just learned to an overview of problems in Geometry. I also review some important techniques that students frequently struggle with involving both Linear and Quadratic Equations.

The Appendix serves as a review of some known topics, but also includes new information that can be considered enrichment for students who may want to learn more. I hope you enjoy using this book as much as I have enjoyed creating it. I believe this is a new and important resource for building the foundation of ongoing excellence in mathematics.

Terri Husted

Because division of fractions can be tricky, we will spend more time reviewing multiplication and division of fractions later in this chapter.

The following are examples of **terms**. Notice that each group or unit is joined by multiplication or division. Also, remember that a constant (a number alone) is also a term.

One Term	Why?
$5 \bullet 8$	5 and 8 are connected by multiplication.
4n	4n means 4 times n.
$\frac{20}{30}$	The division bar creates one term.
25	This is a constant. It's also a term.
$\frac{10 \bullet 3 + 8}{2}$	The fraction (division) bar creates one term

Terms are important because they will help you perform operations correctly. Let's look at the last example shown. You can find the answer to the numerator by multiplying 10 times 3 then adding 8 which equals 38, and then dividing 38 by 2 which equals 19.

Often, students make this mistake: They divide 10 by 2, get 5, multiply 5 by 3 which is 15, then add 8 which gives the wrong answer of 23.

If you remember that the fraction bar is connecting the entire term, it is legal to divide the 10 by 2 AND the 8 by 2 because the dividing bar is creating one term.

$$\frac{\overset{5}{10} \bullet 3 + \overset{4}{8}}{\underset{1}{2}}$$

Since dividing the entire numerator by 2 creates an entire term, both 10 • 3 and 8 are being divided by the denominator of 2. The 10 has a factor of 2 which can be divided by 2 and so does the 8. The answer is 15 + 4 which is 19. Later, we'll learn how to use the distributive property to remove a factor of 2 from the two addends in the numerator in order to simplify the entire term.

We will next review why we multiply 5 • 3 first, then add 4, but before we move on, please review these first two pages carefully as they will give you the foundation you need to perform the correct steps to solve future problems.

Zero and a Negative Exponents

Exponents do not need to be only positive whole numbers. An exponent can be 0 and an exponent can also be a negative number. In fact, an exponent can even be a fraction and we'll look at those in a later chapter. Let's look at this pattern using base 10 and base 2.

Notice as the exponent decreases the answer is divided each time by the base (10).

$$10^4 = 10{,}000$$
$$10^3 = 1000$$
$$10^2 = 100$$
$$10^1 = 10$$
$$10^0 = 1$$
$$10^{-1} = \frac{1}{10}$$
$$10^{-2} = \frac{1}{100}$$

$$2^4 = 16$$
$$2^3 = 8$$
$$2^2 = 4$$
$$2^1 = 2$$
$$2^0 = 1$$
$$2^{-1} = \frac{1}{2}$$
$$2^{-2} = \frac{1}{4}$$

Notice as the exponent decreases the answer is divided each time by the base (2).

Sometimes when looking at the base 10 pattern (on the left) you may think the pattern is "we're just losing a 0 each time," but make sure you truly understand that losing a 0 each time is the result of dividing by the base of 10. To clarify: from 10^0 to 10^{-1} the answer 1 was divided by 10 to get $\frac{1}{10}$ or 0.1, etc.

What you should also notice is that any base (except 0) to the 0 power is always 1. Since we cannot divide by 0 (remember dividing by 0 is not allowed, as it's undefined), we cannot show a pattern as above using a base of 0..

Do you also notice that a negative exponent next to a base results in the reciprocal of that base to the positive power?

$$4^{-3} = \left(\frac{4}{1}\right)^{-3} = \left(\frac{1}{4}\right)^{3} = \frac{1}{64}$$

If a base is positive, a negative power will never make the base turn negative. The negative power just signifies a reciprocal.

Match to Solve the Puzzle

Who was this amazing African-American black mathematician and aerospace engineer who helped NASA get Americans astronauts in space?

1 $(-2)^4$	A) 1
2 5^0	O) −36
3 $\mid -5 -10 \mid$	J) ∅
4 $\left(\frac{6}{5}\right)^{-2}$	N) $\frac{1}{36}$
5 $\frac{0}{15}$	W) 0 (zero)
6 $\frac{15}{0}$	R) 15
7 $\frac{(-12)2^2}{-3 \bullet 16}$	M) 16
8 $\mid -2 \mid - \mid 3 \mid - 5$	K) $\frac{1}{10}$ or 0.1
9 10^{-1}	S) −1
10 $\frac{-100}{100}$	Y) $\frac{25}{36}$
11 $\_\_\_\_ \bullet -2 = 72$	C) −6
12 $(-6)^{-2}$	

__ __ __ __   __   __ __ __ __ __ __ __
1 2 3 4   5   6 7 8 9 10 11 12

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Chapter 4

Working with Terms and Polynomials

In this chapter we will work with expressions that have many terms with numbers and variables. We call some of these expressions polynomials. Poly means "many" and "nomial" means parts or terms. A polynomial may have one or more terms. Not all expressions in math are polynomials.

Below are examples of polynomials. Any polynomial that has more than three terms is just called a polynomial. Notice that a monomial (a one-term polynomial) can be a single constant or a single variable.

The number in front of a variable is called the **coefficient**. A coefficient is a factor of that term. Coefficients can be rational or irrational numbers. Even a variable can be a coefficient if it's acting like a constant in front of another variable (such as the "a" in ax^2).

To be called a polynomial, the expression may only have exponents with whole numbers and no variables in the denominator.

Notice how terms are separated by addition and subtraction signs.

Monomials (1 term)	**Binomial** (2 terms)	**Trinomials** (3 terms)
-5	$n + 5$	$n^2 + 3n - 8$
y	$25n^2 - 100$	$3r^4 - 7r + 7$
$4x^3yz$	$8h^3 - 81$	$6x^4 - 3x + 20$
$.25x^5$	$\frac{1}{4}w - 2$	$.08m^5 + 18m - 1$
$\sqrt{10}$	$w - \sqrt{5}$	$18 - 3x + x^2$

The math expressions below are not called polynomials although we will work with them throughout in this book.

$$\frac{1}{n} \qquad \sqrt{y} \qquad 3x^{-1} \qquad \frac{1}{y+3}$$

Polynomial rules will help you combine expressions with numbers and variables and later will help you become an expert at solving equations and inequalities. Inequalities are sentences with >, ≥, <, ≤.

Factor by Grouping

Factor the following. Follow the steps on the first two problems.

1. $2n^2 + 9n + 9$
   a) What is the first coefficient times the last term? ______
   b) What multiplies to 18 that adds to the coefficient of the middle term?
   ______ and 6.
   c) You can rewrite the problem as $2n^2 + 6n +$ ________ + 9.
   d) It factors now into 2n(________) + 3(________).
   e) What is the final answer? ________________________

2. $2x^2 + 13x - 7$
   a) What is the first coefficient times the last term? ______
   b) What multiplies to −14 that adds to the coefficient of the middle term?
   ______ and 14.
   c) Rewrite the problem as $2x^2 + 14x - x - 7$. This factors into
   $2x(x + 7) - 1($ ________).
   d) What is the final answer? ______________________

3. $3x^2 + 8x + 4$

4. $3n^2 + 14n + 8$

5. $8x^2 - 6x + 1$

6. $3x^2 - x - 14$

7. $2w^2 - 15w + 25$

8. $3y^2 + 23y + 30$

Hint:
Factors of 90

1 • 90	5 • 18
2 • 45	6 • 15
3 • 30	9 • 10

Use this space to check the answers to the following by distributing.

9. Problem #5

10. Problem #8

Equations With Fractions and Decimals

In the last chapter, we learned how to combine fractional expressions by finding the least common denominator (LCD). We are now ready to use the LCD to get rid of fractions entirely when we're working with equations.

Ex 1: $\frac{x}{24} + \frac{2x+1}{8} = \frac{x}{3}$

Find the LCD of all the terms. In this case it is 24.

$$\overset{1}{\cancel{24}} \bullet \frac{x}{\cancel{24}} + \overset{3}{\cancel{24}} \bullet \frac{2x+1}{\cancel{8}} = \overset{8}{\cancel{24}} \bullet \frac{x}{\cancel{3}}$$

Multiply each term by the LCD.

Now cancel: 24 ÷ 24 = 1, 24 ÷ 8 =3, and 24 ÷ 3 = 8. Multiply by each term.

$$1x + 3(2x + 1) = 8x$$

$$x + 6x + 3 = 8x$$

$$\cancel{7x} + 3 = 8x$$

$$\cancel{-7x} \qquad -7x$$

$$3 = x \quad \text{or} \quad x = 3$$

Notice the middle term requires a parenthesis since the entire term was being divided by 8.

Check $\frac{3}{24} + \frac{2(3)+1}{8} = \frac{3}{3}$?

$\frac{1}{8} + \frac{7}{8} = 1$? Yes!

Ex 2: $2 + \frac{x}{4} = \frac{2x-5}{7}$

Find the LCD of all the terms. It's 28.

Multiply each term by the LCD.

$$\overset{28}{\cancel{28}} \bullet \frac{2}{1} + \overset{7}{\cancel{28}} \bullet \frac{x}{\cancel{4}} = \overset{4}{\cancel{28}} \bullet \frac{2x-5}{\cancel{7}}$$

Cancel and multiply each term.

$$56 + 7x = 4(2x - 5)$$

Notice the term on the right side of the equation needs a parenthesis.

Continue to solve.

$$56 + \cancel{7x} = 8x - 20$$

$$\cancel{-7x} \qquad -7x$$

$$56 = x \cancel{- 20}$$

$$+20 \qquad \cancel{+20}$$

$$76 = x \quad \text{or} \quad x = 76$$

Check $2 + \frac{76}{4} = \frac{2(76)-5}{7}$?

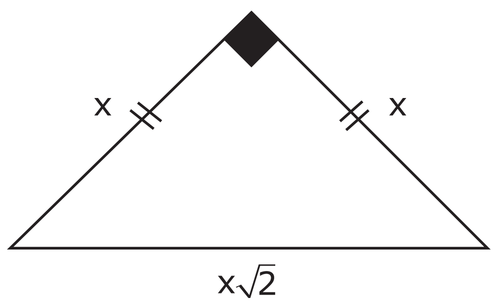
$2 + 19 = \frac{152-5}{7}$?

$21 = 21$ Yes!

Special Right Triangles

The following right triangles are important to know as they provide a shortcut for finding the missing side.

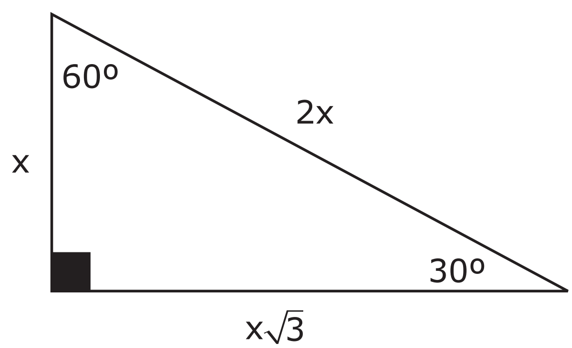
The **45º-45º-90º** triangle shown here is an isosceles triangle (2 sides are congruent and two angles are congruent). All isosceles right triangles are similar to each other and therefore the sides have the same proportions shown here.



Remember "similar" means the corresponding angles are congruent and the corresponding sides are in proportion.

If you know one side, the hypotenuse will always be the measurement of one side times the square root of 2.

The **30º-60º-90º** triangle shown here has the following property. The hypotenuse will always be twice the short side, and the middle side is the short side times the square root of 3. Remember all 30º-60º-90º triangles no matter how large or small will be similar. Therefore their sides will have the same proportions shown here:



These two special right triangles are important in geometry and especially later on, in trigonometry. Having some practice with them will help you develop more practice with square roots and give you more confidence in math.

Chapter 9

Understanding Functions

What is a Function?

A function in math is the relation or rule between two numbers where an input (x) results each time in the exact same output (y) for that input. The input is called the independent variable and the output is the dependent variable (as y depends on what you let x equal).

FUNCTION

$-2 \rightarrow$	-8
$-1 \rightarrow$	-5
$0 \rightarrow$	2
$1 \rightarrow$	1
$2 \rightarrow$	4
$3 \rightarrow$	7

FUNCTION

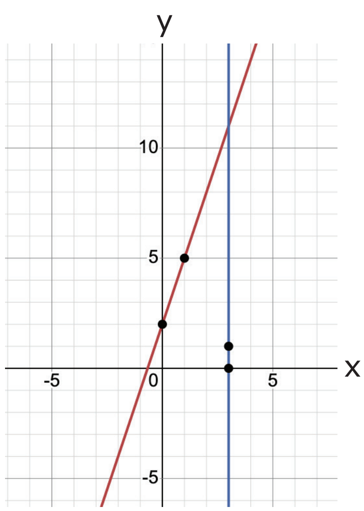
$-2 \rightarrow$	4
$-1 \rightarrow$	1
$0 \rightarrow$	0
$1 \rightarrow$	1
$2 \rightarrow$	4
$3 \rightarrow$	9

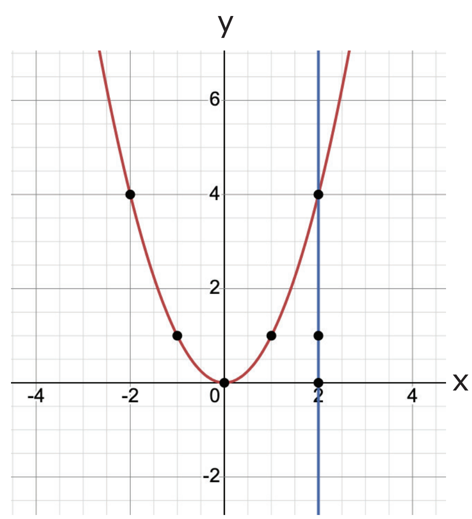
NOT A FUNCTION

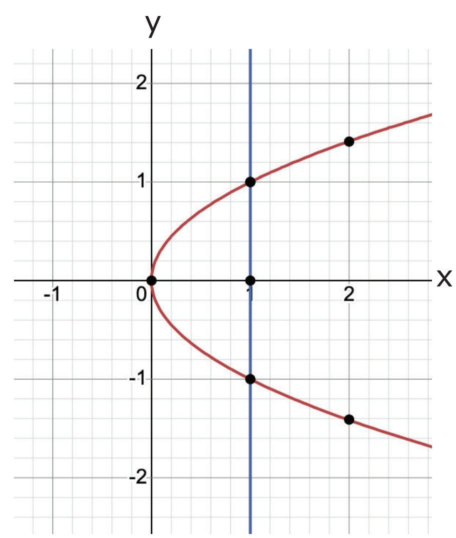
$-2 \rightarrow$	0
$0 \rightarrow$	1.5
$\searrow$	1.5
$2 \rightarrow$	2
$\searrow$	-2
$3 \rightarrow$	9
$\searrow$	-9

Notice that it's fine for a different input to give you the same output as another input such as in (−1, 1) and (1, 1), but it's not a function if a given input gives you two different outputs as in (0, 1.5) and (0, −1.5).

Another way to tell if you have a function is to look at its graph and do what is called a vertical line test. Draw a vertical line across a graph (blue line below) and if there are NO points above or below another point, then the relation is a function.







Here, you see many points above and below the blue line, so this is NOT a function.

Finding the Equation from a Table of Values

If you're given a table of values for a linear equation, you can find the slope by finding the rate of change, how the y's are changing. But only if the x values are in sequential order. Let's review this concept:

x	2	3	4	5
y	0	−2	−4	−6

To find the slope, first make sure the x's are one unit apart and in order. Next, take a look at how the y's are changing. As x increases, the y's are decreasing by 2, so the rate of change or the slope is −2.

To find the y-intercept, you need to know the value that corresponds when x = 0. That is not shown on the table above, but can you figure it out? You have (3, −2) and to the left (2, 0), so continuing the table to the left, you would have (1, 2) and then (0, 4).

x	0	1	2	3	4	5
y	4	2	0	−2	−4	−6

The slope is −2 and the y-intercept is 4. So, the equation is $y = -2x + 4$.

You can also find the slope when given two points. Use any two points on your table, for example, (3, −2) and (4, −4) and use the slope formula below.

Slope formula

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Δ delta symbol means "change."

So, substituting (3, −2) and (4, −4) in the slope formula: $\frac{(-4) - (-2)}{(4) - (3)} = \frac{-2}{1}$

or −2 as we found above. Make sure when you use the slope formula, that you use parentheses to help you avoid errors with negative signs.

Dividing a Polynomial by a Binomial by Using Synthetic Division

Synthetic division is based on long division of polynomials. It can be a lot simpler than long division with variables, etc.

Suppose you need to know if (x – 5) is a factor of the trinomial $2x^2 - 13x + 15$. Here's what you do. If (x – 5) is a factor then 5 would be a root.

Write 5 on the outside and put the <u>coefficients</u> of your trinomials in order as below.

$$\begin{array}{r|rrr} 5 & 2 & -13 & 15 \\ \hline & & 10 & -15 \\ \hline & \mathbf{2} & \mathbf{-3} & 0 \end{array}$$

Drop the first coefficient down.
Multiply the 5 times 2 and write it under the –13.
Add –13 + 10 = –3

Multiply 5 times –3 = –15

Now add 15 + (–15) = 0

Getting zero means it is a factor. The coefficients of the other factor are in bold. You will write 2x – 3. Notice the degree of the x is one less than the degree of the trinomial you started with.

Ex 2: Is (x + 8) a factor of $3x^2 + 23x - 8$? Another way of asking this is: Is $3x^2 + 23x - 8$ divisible by (x + 8)? If so, –8 would be a root.

$$\begin{array}{r|rrr} -8 & 3 & 23 & -8 \\ \hline & & -24 & 8 \\ \hline & \mathbf{3} & \mathbf{-1} & 0 \end{array}$$

Yes, it is a factor. The coefficients of the other factor are in bold. You will write (3x – 1). Again, the degree of the x term is one less than the degree of the trinomial you started with.

In the next example, you will have a remainder and find out what to do with it.