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Putting All the Steps Together

Now, we're using all the steps because the equations are longer, but not necessarily harder.

Steps for Solving Equations

- 1 ▶ **Distribute.**
- 2 ▶ **Combine** all like terms on each side.
- 3 ▶ **Move** variables to the same side.
- 4 ▶ **Undo** + and - (additive inverse).
- 5 ▶ **Undo** · and ÷ (multiplicative inverse or reciprocal).

1 $10(x - 1) + 20 = 5(x - 3)$

$$\begin{array}{r}
 10(x - 1) + 20 = 5(x - 3) \\
 10x - 10 + 20 = 5x - 15 \\
 10x + 10 = 5x - 15 \\
 -5x \qquad -5x \\
 \hline
 5x + 10 = -15 \\
 -10 \qquad -10 \\
 \hline
 5x = -25 \\
 \frac{5x}{5} = \frac{-25}{5} \\
 x = -5
 \end{array}$$

- 1 ▶ Distribute.
- 2 ▶ Combine like terms on each side.
- 3 ▶ Move variables to the same side.
- 4 ▶ Undo + and -.
- 5 ▶ Undo · and ÷.

2 $18 - 5(x - 1) = -10 + 3$

$$\begin{array}{r}
 18 - 5x + 5 = -7 \\
 23 - 5x = -7 \\
 -23 \qquad -23 \\
 \hline
 -5x = -30 \\
 \frac{-5x}{-5} = \frac{-30}{-5} \\
 x = 6
 \end{array}$$

- 1 ▶ Distribute.
- 2 ▶ Combine like terms on each side.
- 3 ▶ Step 3 is not needed here.
- 4 ▶ Undo + and -.
- 5 ▶ Undo · and ÷.

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Try to remember that there are many ways to solve a word problem so be ready to justify your method and be open to listen to how others solved the same problem.

- 2 In five more years, Lucia’s grandfather will be eight times as old as Lucia was two years ago. When you add their present ages the sum is 69 years. How old is each one now?

	Two Years Ago	Now	In Five Years
Lucia	$x - 2$	x	$x + 5$
Grandpa	$8(x - 2) - 5 - 2$	$8(x - 2) - 5$	$8(x - 2)$

Steps 1-3: Read, create chart, fill in given information.

Step 4: Complete the chart. Add five years and subtract two years.

Equation: $x + 8(x - 2) - 5 = 69$
 $x + 8x - 16 - 5 = 69$
 $9x - 21 = 69$
 $x = 10$

Step 5: Use another sentence relating the boxes of the chart to write the equation. Present ages add to a sum of 69 years.

Solving the equation.

Lucia is 10 years old, her grandfather is 59 years old

Answer the following questions using a chart. Charts for 1-5 are provided.

- 1 Sokhem is Chenda’s older brother. In six more years Sokhem will be twice Chenda’s age now. In six more years the sum of their ages then will be 60 years. How old is each now?

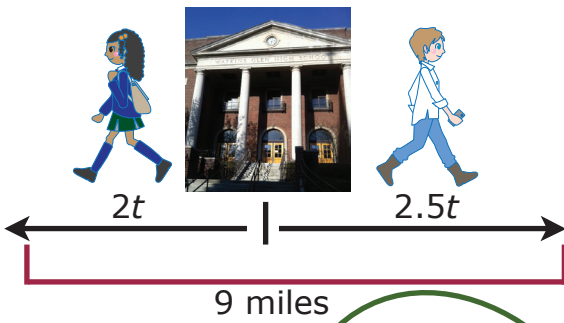
	Now	In Six Years
Chenda	x	
Sokhem		$2x$

Now follow step 4.

Toward and Away Problems

- 1 Dani and Netta leave school at the same time and travel in opposite directions. Netta walks at the rate of 2 mph and Dani walks at the rate of 2.5 mph. After how much time would they be 9 miles apart if they kept walking at those rates?

Draw a picture.



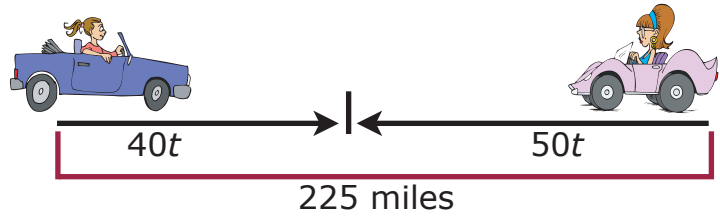
Now make a chart.

	Rate (mph)	Time	Distance
Netta	2	t	$2t$
Dani	2.5	t	$2.5t$
Total			9

t stands for time.
Distance Netta covered.
Distance Dani covered.
Total distance.

Equation: $2t + 2.5t = 9$
 $4.5t = 9$ *Solving the equation.*
 $t = 2$ hours

- 2 Franny and Kate are 225 miles apart and are driving toward each other. Franny is going 40 mph and Kate is going 50 mph. If both leave at the same time, how long does it take for them to meet?



Equation: $40t + 50t = 225$
 $90t = 225$
 $t = 2.5$ hours


	Rate (mph)	Time	Distance
Franny	40	t	$40t$
Kate	50	t	$50t$
Total			225

Total distance.

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Subtraction of Polynomials

If you remember from Chapter 1, subtraction is the same as adding the additive inverse. You also need to remember the key words from, less, and less than. Just as in addition, you can only combine like terms.

$$1 \quad (5x - 3) - (2x + 5)$$



The subtraction sign is in the middle. You must change the signs of every term that follows that subtraction sign.

Rewrite: $5x - 3 + (-2x - 5)$

Answer: $3x - 8$

$$2 \quad \text{Take } (-3ab - b + 2) \text{ from } (-2ab + 2b - 5).$$

$$\text{Rewrite: } (-2ab + 2b - 5) - (-3ab - b + 2)$$

$$(-2ab + 2b - 5) + (3ab + b - 2)$$


Remember that the words from and less than reverse the order.

Answer: $ab + 3b - 7$

$$3 \quad \text{What is } (-15 + 2b^2) \text{ less than } (8b^2 + b - 4)?$$


$$\text{Rewrite: } (8b^2 + b - 4) - (-15 + 2b^2)$$

$$(8b^2 + b - 4) + (15 - 2b^2)$$

Answer: $6b^2 + b + 11$

One suggestion to help with distributing the subtraction sign is to put a **1** (so you have -1) in front of what you're about to subtract.

$$4 \quad (8a^2b + 2c - 4) - (-12a^2b - 4c + 6)$$

$$\text{Rewrite } (8a^2b + 2c - 4) - \mathbf{1}(-12a^2b - 4c + 6)$$


Distribute: $8a^2b + 2c - 4 + 12a^2b + 4c - 6$

Answer: $20a^2b + 6c - 10$

Polynomial Thinking Questions

Answer these questions.

1 What are two ways that you can explain why $\frac{b^4}{b^4} = 1$ (if $b \neq 0$)?

2 When Lisa simplified this problem $\frac{16a - 8}{16}$ she got "a - 2." What did Lisa do wrong?

3 If 10^{-2} is the same as $\frac{1}{10^2}$, how would you write "40⁻²"? Explain your thinking.

4 What do we call these binomials? $(x^2 - 25)$; $(m^2 - 49)$; $(y^2 - 1)$? Hint: See p. 153.

5 When Larry simplified this expression $\frac{3d^4 - d}{d}$, he got $3d^3$. What did Larry do wrong? What is the correct answer?

6 Show by writing the factors out why $x^4 \div x^6 = \frac{1}{x^2}$.

7 Why is $(3^3)(3^2)$ NOT 9^5 ? Explain your thinking.

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8 Min-ji has 10 skeins of yarn, each one 220 yards long. She wants to make scarves and hats for presents. Each scarf takes 190 yards of yarn, and a hat takes 150 yards.

a Write an inequality to represent the possible scarves and hats she could knit.



b Using graph paper or a graphing calculator, graph the inequality. You will need to change the window settings. Hint: Use $X_{min} = 0$, $X_{max} = 20$, $Y_{min} = 0$, $Y_{max} = 15$.

In a graphing calculator, type your equation using fractions. To change a decimal to a fraction in a graphing calculator: When the decimal is on the screen, go to **MATH**, **FRAC**, then **ENTER**.

c If Min-ji decided to make only scarves, what is the greatest number of scarves that she could knit?

d If Min-ji decided to make only hats, what is the greatest number of hats that she could knit?

e If Min-ji sold her scarves for \$8 and her hats for \$5, with the yardage she has, would she make more money selling only scarves, only hats, or both? Explain your thinking.
