

## Table of Contents

*Arithmetic* (a poem) ..... ii

About the Author ..... vi

**What Is Algebra?** ..... **1**

**Chapter 1 Our Number System  
Sets, Operations, and Properties** ..... **2-29**

Sets and Set Notation ..... 2

The Number Line and Graphing Sets ..... 9

Operations and Properties of Rational Numbers ..... 14

    Addition of Rationals ..... 14

    Subtraction of Rationals ..... 16

    Multiplication and Division of Rationals ..... 17

Order of Operations ..... 22

The Closure Property ..... 27

**Chapter 2 Evaluating Expressions and Solving Equations** ..... **30-58**

What Are Expressions, Equations, and Inequalities? ..... 30

Evaluating Expressions ..... 31

Equations – A Balancing Act! ..... 33

Steps for Solving Equations ..... 35

    Solving Simple Equations Using the Additive Inverse ..... 36

    Solving Simple Equations Using the Multiplicative Inverse ..... 37

    Solving Two-Step Equations ..... 39

    Moving Variables to the Same Side ..... 43

    Putting All the Steps Together ..... 45

Equations With Decimals and Fractions ..... 51

    Calculator Basics ..... 51

Working With Formulas ..... 54

**Chapter 3 From Words to Algebra  
Translating and Solving Word Problems** ..... **60-84**

Translating Expressions and Equations ..... 60

Strategies for Solving Word Problems ..... 62

Diagramming Geometry Word Problems ..... 63

Word Problems About Missing Numbers ..... 67

Consecutive Integer Problems ..... 69

Coin Problems ..... 73

Age Problems ..... 78

Free resource from www.criticalthinking.com. Commercial redistribution prohibited

<b>Chapter 4</b>	<b>From Words to Algebra More Word Problems</b>	<b>85-110</b>
	Percent and Ratio Problems	85
	Percent Increase and Decrease	88
	Relative Error and Percent Error	89
	Mixture Problems	92
	Motion Problems	97
	Distance Problems	98
	Toward and Away Problems	99
	Chase Problems	102
	Work Problems	107
<b>Chapter 5</b>	<b>Inequalities, Compound Inequalities, and Absolute Value</b>	<b>111-131</b>
	Understanding How to Solve Inequalities	112
	Solving Inequality Word Problems	117
	Solving Compound Inequalities	120
	Solving Absolute Value Equations	124
	Absolute Value Inequalities	127
<b>Chapter 6</b>	<b>Polynomials</b>	<b>132-162</b>
	What Is a Polynomial?	132
	Addition of Polynomials	136
	Subtraction of Polynomials	141
	Exponents and Multiplication of Monomials	143
	Multiplication of a Polynomial by a Monomial	146
	Multiplication of Binomials	148
	Special Products	153
	Division of Monomials	156
	Understanding Negative Exponents	158
	Dividing Polynomials by Monomials	160
<b>Chapter 7</b>	<b>Factoring</b>	<b>163-185</b>
	The Greatest Common Factor	163
	Factoring Rules	168
	Factoring Trinomials With a Leading Coefficient	172
	Solving Quadratic Equations	177
	Word Problems With Quadratic Equations	180
	Quadratic Equations and the Pythagorean Theorem	183
<b>Chapter 8</b>	<b>Working With Radicals</b>	<b>186-205</b>
	Simplifying Square Roots	190
	Adding and Subtracting Square Roots	192
	Multiplying Square Roots	194
	Dividing Square Roots	197
	The Quadratic Formula	203

<b>Chapter 9</b>	<b>Linear Functions</b>	<b>206-234</b>
	What Is a Function?	206
	Linear Functions	210
	What Is Slope?	216
	Zero Slope and Undefined Slope	218
	Writing Equations for Linear Functions	221
	Point-Slope Formula	225
	Linear Function Applications	228
	Graphing Linear Functions on a Calculator	229
	Scatterplots	231
	Types of Correlations	233
<b>Chapter 10</b>	<b>Systems of Equations and Inequalities</b>	<b>235-263</b>
	Method 1 – Graphing	235
	Method 2 – Elimination	239
	Method 3 – Substitution	243
	Using Systems to Solve Real-Life Problems	245
	Graphing Inequalities	249
	Systems of Inequalities	255
<b>Chapter 11</b>	<b>Other Types of Functions</b>	<b>264-294</b>
	Absolute Value Functions	264
	Quadratic Functions	270
	Vertex Form of a Quadratic Function	276
	Applications of Parabolas	280
	Exponential Functions	284
	Exponential Growth and Decay	285
	Thinking About Patterns – A Helpful Guide	289
	Graphing a System of Functions	293
<b>Chapter 12</b>	<b>Working With Algebraic Fractions</b>	<b>295-312</b>
	Multiplying and Dividing Algebraic Fractions	300
	Adding and Subtracting Algebraic Fractions	303
	Solving Equations With Algebraic Fractions	307
	Revisiting Work Problems	311
	<b>Thinking About Algebra</b>	<b>313</b>
	<i>Algebra</i> (a poem)	314
	<b>Glossary</b>	<b>315-318</b>
	<b>Answers</b>	<b>319-354</b>
	Reference	355
	Graphing Paper	356-359

Free resource from www.criticalthinking.com. Commercial redistribution prohibited

## Putting All the Steps Together

Now, we're using all the steps because the equations are longer, but not necessarily harder.

### Steps for Solving Equations

- 1 ▶ **Distribute.**
- 2 ▶ **Combine** all like terms on each side.
- 3 ▶ **Move** variables to the same side.
- 4 ▶ **Undo** + and - (additive inverse).
- 5 ▶ **Undo** · and ÷ (multiplicative inverse or reciprocal).

1  $10(x - 1) + 20 = 5(x - 3)$

$$\begin{array}{r}
 \begin{array}{r}
 \text{↖ ↗} \quad \text{↖ ↗} \\
 10(x - 1) + 20 = 5(x - 3) \\
 10x - 10 + 20 = 5x - 15 \\
 10x + 10 = 5x - 15 \\
 \underline{-5x} \quad \quad \underline{-5x} \\
 5x + 10 = -15 \\
 \quad \underline{-10} \quad \quad \underline{-10} \\
 \hline
 5x = -25 \\
 \underline{\quad \quad \quad \div 5} \\
 x = -5
 \end{array}
 \end{array}$$

- 1 ▶ Distribute.
- 2 ▶ Combine like terms on each side.
- 3 ▶ Move variables to the same side.
- 4 ▶ Undo + and -.
- 5 ▶ Undo · and ÷.

2  $18 - 5(x - 1) = -10 + 3$

$$\begin{array}{r}
 18 - 5x + 5 = -7 \\
 \text{↖ ↗} \\
 23 - 5x = -7 \\
 \underline{-23} \quad \quad \underline{-23} \\
 \hline
 -5x = -30 \\
 \underline{\quad \quad \quad \div -5} \\
 x = 6
 \end{array}$$

- 1 ▶ Distribute.
- 2 ▶ Combine like terms on each side.
- 3 ▶ Step 3 is not needed here.
- 4 ▶ Undo + and -.
- 5 ▶ Undo · and ÷.

Free resource from www.criticalthinking.com. Commercial redistribution prohibited

Try to remember that there are many ways to solve a word problem so be ready to justify your method and be open to listen to how others solved the same problem.

- 2 In five more years, Lucia’s grandfather will be eight times as old as Lucia was two years ago. When you add their present ages the sum is 69 years. How old is each one now?

	Two Years Ago	Now	In Five Years
Lucia	$x - 2$	$x$	$x + 5$
Grandpa	$8(x - 2) - 5 - 2$	$8(x - 2) - 5$	$8(x - 2)$

Steps 1-3: Read, create chart, fill in given information.

Step 4: Complete the chart. Add five years and subtract two years.

Equation:  $x + 8(x - 2) - 5 = 69$   
 $x + 8x - 16 - 5 = 69$   
 $9x - 21 = 69$   
 $x = 10$

Step 5: Use another sentence relating the boxes of the chart to write the equation. Present ages add to a sum of 69 years.

Solving the equation.

Lucia is 10 years old, her grandfather is 59 years old

Answer the following questions using a chart. Charts for 1-5 are provided.

- 1 Sokhem is Chenda’s older brother. In six more years Sokhem will be twice Chenda’s age now. In six more years the sum of their ages then will be 60 years. How old is each now?

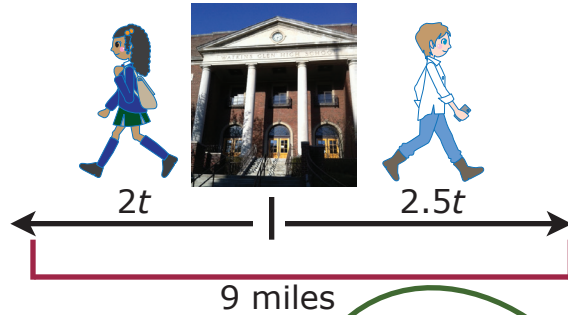
	Now	In Six Years
Chenda	$x$	
Sokhem		$2x$

Now follow step 4.

### Toward and Away Problems

- 1 Dani and Netta leave school at the same time and travel in opposite directions. Netta walks at the rate of 2 mph and Dani walks at the rate of 2.5 mph. After how much time would they be 9 miles apart if they kept walking at those rates?

Draw a picture.



Now make a chart.

	Rate (mph)	Time	Distance
Netta	2	$t$	$2t$
Dani	2.5	$t$	$2.5t$
Total			9

$t$  stands for time.

Distance Netta covered.

Distance Dani covered.

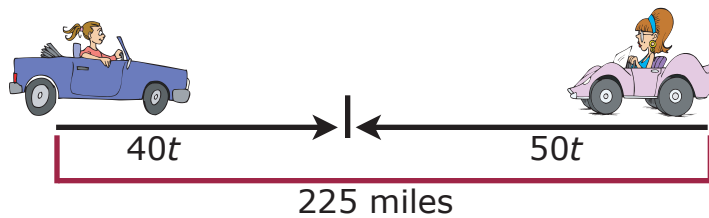
Total distance.

Equation:  $2t + 2.5t = 9$

$4.5t = 9$  Solving the equation.

$t = 2$  hours

- 2 Franny and Kate are 225 miles apart and are driving toward each other. Franny is going 40 mph and Kate is going 50 mph. If both leave at the same time, how long does it take for them to meet?



Equation:  $40t + 50t = 225$

$90t = 225$

$t = 2.5$  hours

	Rate (mph)	Time	Distance
Franny	40	$t$	$40t$
Kate	50	$t$	$50t$
Total			225

Total distance.

## Subtraction of Polynomials

If you remember from Chapter 1, subtraction is the same as adding the additive inverse. You also need to remember the key words from, less, and less than. Just as in addition, you can only combine like terms.

**1**  $(5x - 3) - (2x + 5)$

The subtraction sign is in the middle. You must change the signs of every term that follows that subtraction sign.

Rewrite:  $5x - 3 + (-2x - 5)$

Answer:  $3x - 8$

**2** Take  $(-3ab - b + 2)$  from  $(-2ab + 2b - 5)$ .

Rewrite:  $(-2ab + 2b - 5) - (-3ab - b + 2)$   
 $(-2ab + 2b - 5) + (3ab + b - 2)$

Remember that the words from and less than reverse the order.

Answer:  $ab + 3b - 7$

**3** What is  $(-15 + 2b^2)$  less than  $(8b^2 + b - 4)$ ?

Rewrite:  $(8b^2 + b - 4) - (-15 + 2b^2)$   
 $(8b^2 + b - 4) + (15 - 2b^2)$

Answer:  $6b^2 + b + 11$

One suggestion to help with distributing the subtraction sign is to put a **1** (so you have -1) in front of what you're about to subtract.

**4**  $(8a^2b + 2c - 4) - (-12a^2b - 4c + 6)$

Rewrite  $(8a^2b + 2c - 4) - \mathbf{1}(-12a^2b - 4c + 6)$

Distribute:  $8a^2b + 2c - 4 + 12a^2b + 4c - 6$

Answer:  $20a^2b + 6c - 10$

## Polynomial Thinking Questions

Answer these questions.

1 What are two ways that you can explain why  $\frac{b^4}{b^4} = 1$  (if  $b \neq 0$ )?

---



---

2 When Lisa simplified this problem  $\frac{16a - 8}{16}$  she got "a - 2." What did Lisa do wrong?

---



---

3 If  $10^{-2}$  is the same as  $\frac{1}{10^2}$ , how would you write "40<sup>-2</sup>"? Explain your thinking.

---



---

4 What do we call these binomials?  $(x^2 - 25)$ ;  $(m^2 - 49)$ ;  $(y^2 - 1)$ ? Hint: See p. 153.

---



---

5 When Larry simplified this expression  $\frac{3d^4 - d}{d}$ , he got  $3d^3$ . What did Larry do wrong? What is the correct answer?

---



---

6 Show by writing the factors out why  $x^4 \div x^6 = \frac{1}{x^2}$ .

---



---

7 Why is  $(3^3)(3^2)$  NOT  $9^5$ ? Explain your thinking.

---



---

Free resource from www.criticalthinking.com. Commercial redistribution prohibited



8 Min-ji has 10 skeins of yarn, each one 220 yards long. She wants to make scarves and hats for presents. Each scarf takes 190 yards of yarn, and a hat takes 150 yards.

a Write an inequality to represent the possible scarves and hats she could knit.



b Using graph paper or a graphing calculator, graph the inequality. You will need to change the window settings. Hint: Use  $X_{min} = 0$ ,  $X_{max} = 20$ ,  $Y_{min} = 0$ ,  $Y_{max} = 15$ .

In a graphing calculator, type your equation using fractions. To change a decimal to a fraction in a graphing calculator: When the decimal is on the screen, go to **MATH**, **FRAC**, then **ENTER**.

c If Min-ji decided to make only scarves, what is the greatest number of scarves that she could knit?

d If Min-ji decided to make only hats, what is the greatest number of hats that she could knit?

e If Min-ji sold her scarves for \$8 and her hats for \$5, with the yardage she has, would she make more money selling only scarves, only hats, or both? Explain your thinking.

---



---



---



---