

Table of Contents

Introduction	ii
Chapter 1 Family of Numbers	1-14
Family of Numbers	1
Important Divisibility Rules.....	3
Prime Factorization.....	5
The Rats! Rational Numbers	6
Important Factors	10
Chapter Review	13
Chapter 2 Working With Integers	15-31
Addition of Integers	15
Absolute Value.....	16
Adding Positive and Negative Integers	17
Subtraction of Integers	21
Multiplication and Division of Integers.....	26
Working With Exponents	28
Chapter Review	30
The Cowboy Story.....	31
Chapter 3 Working With Rational Numbers	32-48
Adding and Subtracting Rational Numbers	32
Working With Different Denominators	36
Multiplication and Division of Positive and Negative Rationals	38
Order of Operations.....	42
Understanding Why Order of Operations Is Important.....	45
Chapter Review	47
Chapter 4 Ratio, Proportion, and Percent	49-76
Converting Fractions to Decimals and Terminating Decimals to Fractions.....	49
Common Fractions/Decimals	50
How to Tell if a Fraction Terminates or Repeats	53
Ratio, Rate, and Unit Rate	54
Proportions	59
Scale Drawings	63
Understanding Percent.....	67
Working With Percent	70
Chapter Review	74

Chapter 5	Percent Applications.....	77-86
	Going Shopping- Tax and Tips.....	77
	Percent Increase and Decrease	81
	Understanding Simple Interest vs Compound Interest.....	82
	Chapter Review	85
Chapter 6	Algebraic Expressions.....	87-116
	Expression vs Equations.....	87
	Evaluating Expressions	88
	Translating Expressions.....	90
	Understanding "Less," "Less Than," and "From"	91
	Polynomials.....	94
	Understanding "Like Terms".....	95
	Subtracting Polynomials.....	99
	Multiplication of Monomials - Exploring Exponent Rules	101
	Thinking More About Monomials.....	102
	Powers	104
	Division of Monomials.....	106
	Understanding Negative Powers	108
	Scientific Notation.....	110
	Using Scientific Notation to Simplify	112
	Chapter Review	114
Chapter 7	Equations and Solving Word Problems	117-152
	Properties of Real Numbers (Part 1)	117
	Properties of Real Numbers (Part 2)	120
	Properties of Equality	124
	Solving Algebra Equations.....	126
	Step 4: Using the Additive Inverse	126
	Step 5: Using the Multiplicative Inverse	128
	Solving Two Step Equations.....	131
	Solving Word Problems Using Equations.....	133
	Steps 3, 4, and 5: Moving Variable Terms to the Same Side.....	135
	Steps 2-5: Combining Like Terms on Each Side	137
	Steps 1-5: Using All the Steps to Solve Equations	140
	Consecutive Integer Problems.....	143
	Thinking More About Equations.....	146
	Chapter Review	148
Chapter 8	Inequalities and Applications	153-171
	Graphing Inequalities	153
	Solving Inequalities.....	157
	How to Tell an Equation From an Inequality in Word Problems.....	161
	Understanding Compound Inequalities	165
	Graphing Compound Inequalities.....	166
	Chapter Review	169

Chapter 9	Understanding Square Roots and Irrational Numbers.....	172-195
	Repeating Decimals Cannot Be Irrational Numbers	172
	Squaring and Square Roots	174
	Cube Roots and Other Roots	177
	Simplifying Square Roots	179
	A Look at Irrational PI	182
	The Pythagorean Theorem	185
	Chinese Proof of the Pythagorean Theorem	190
	Chapter Review	191
Chapter 10	Two Dimensional Geometry	196-234
	Naming Angles	196
	Parallel Lines and Transversals	200
	Perimeter.....	205
	Polygons.....	206
	Angles of Triangles and Polygons.....	210
	Types of Triangles	213
	Areas of Parallelograms, Triangles, and Trapezoids.....	216
	Area of Parallelograms	218
	Area of Triangles.....	219
	Area of Trapezoids	220
	Using Algebra to Solve Geometry Problems	222
	Finding the Circumference and Area of a Circle	224
	Chapter Review	228
Chapter 11	Understanding Volume and Surface Area.....	236-251
	Volume of Prisms	236
	Volume of Pyramids.....	239
	Volume of Cylinders and Cones	241
	Surface Area	244
	Surface Area of a Cone	247
	Volume and Surface Area of a Sphere	248
	Chapter Review	249
Chapter 12	Graphing on the Coordinate Plane	252-275
	Graphing Linear Equations	253
	Special Graphs	254
	Using Slope and Y-Intercept to Graph Linear Equations	256
	Types of Slopes	258
	Finding Slope Given Two Points	261
	Writing an Equation Given Two Points	263
	Graphs of Proportional Relationships.....	265
	Systems of Equations	269
	Chapter Review	272

Chapter 13 Transformations and Congruency	276-302
Translations	276
Reflections	280
Rotations	283
Understanding Triangle Congruency (SSS, SAS, ASA).....	286
How to Prove Triangles Are Congruent	290
Dilations	292
Similarity	295
Chapter Review	298
Chapter 14 Understanding Functions.....	303-322
Understanding Functions.....	303
Quadratic Functions	306
Understanding Exponential Functions.....	309
Exponential Growth and Decay	312
Absolute Value Functions	315
Linear, Exponential, and Quadratic Functions Review.....	318
Chapter Review	319
Chapter 15 Probability and Statistics.....	323-358
Understanding OR and AND.....	326
Understanding Many-Stage Experiments.....	328
The Counting Principle	332
Permutations.....	333
Understanding Permutations and Combinations.....	335
Empirical vs Theoretical Probability.....	337
Understanding Statistics	338
Measures of Central Tendency.....	338
Quartiles and Percentiles.....	341
Frequency Tables and Histograms	344
Stem and Leaf Plots	349
Box and Whisker Plots	350
Understanding Surveys.....	353
Chapter Review	355
Final Examination.....	359-367
Glossary	368-372
Reference Sheet	373
Table of Square Roots 1-120	374
Answers	375

Chapter 4

Ratio, Proportion, and Percent

Fractions, decimals, and percents are different ways of representing the same quantities. For some situations, you'll know whether it's better to represent a quantity by a fraction, a decimal, or a percent. Let's review how to change fractions to decimals and terminating decimals to fractions.

Remember from Chapter 1 that any fraction can be changed to a terminating (a decimal that stops) or to a repeating decimal.

Converting Fractions to Decimals and Terminating Decimals to Fractions

Example 1: Change $\frac{3}{8}$ to a decimal.

The fraction bar means "divide." You're dividing 3 by 8. You can think of the 3 sitting on a chair and falling into the box. Since 8 cannot go into 3, put a decimal point after the 3 and add a few zeros. Then start dividing and keep dividing until the decimal terminates or repeats.

$$\begin{array}{r} 0.375 \\ 8 \overline{) 3.000} \\ \underline{- 24} \\ 60 \\ \underline{- 56} \\ 40 \\ \underline{- 40} \\ 0 \end{array}$$

Example 2: Change .375 to a fraction.

The 5 sits on the thousandths place, so write 375 as the numerator and 1,000 as the denominator and reduce. Then divide the fraction by 5. If the GCF (which is 125) was found, the fraction could have been reduced in one step.

$$\frac{375}{1,000} = \frac{75}{200} = \frac{15}{40} = \frac{3}{8}$$

Example 3: Change $-5\frac{2}{3}$ to a decimal.

Since -5 is an integer, you just need to find out what $\frac{2}{3}$ is as a decimal. Again, drop the 2 into the box and divide.

$$\begin{array}{r} 0.666\dots \\ 3 \overline{) 2.000} \\ \underline{- 18} \\ 20 \\ \underline{- 18} \\ 20 \\ \underline{- 18} \\ 0 \end{array}$$

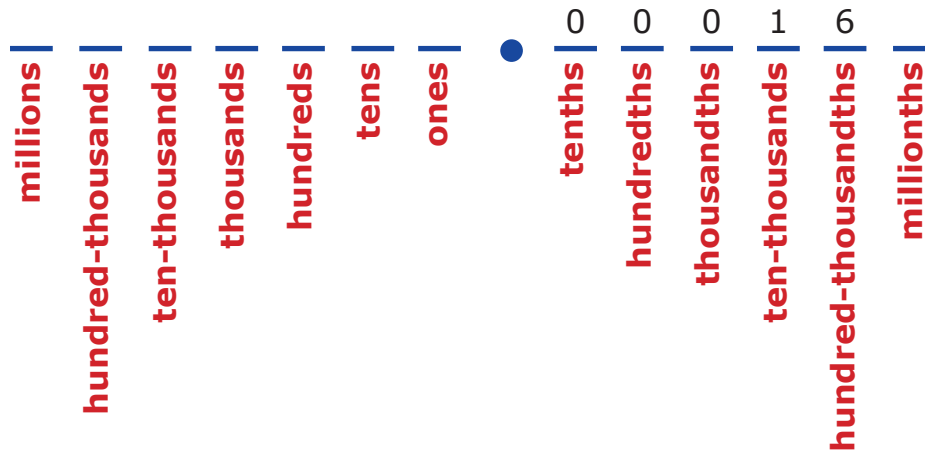
You can see that the 6 will repeat, so the answer is $-5.\overline{6}$. The bar on top of the 6 indicates that the 6 repeats. And yes, you can write your answer as $-5.\overline{66}$.

Common Fractions/Decimals

You can always use the above methods to convert fractions to decimals and terminating decimals to fractions. However, there are 10 common fractions/decimals that appear in many real life situations, as well as in many standardized tests that you might take in the future. It is a great advantage to memorize these. Making flashcards may be useful. Make sure you do not memorize these unless you understand the concept.

$\frac{1}{2} = .5$	$\frac{3}{8} = .375$
$\frac{1}{4} = .25$	$\frac{5}{8} = .625$
$\frac{3}{4} = .75$	$\frac{7}{8} = .875$
$\frac{1}{5} = .2$	$\frac{1}{3} = \overline{.3}$
$\frac{1}{8} = .125$	$\frac{2}{3} = \overline{.6}$

$\frac{1}{5} = .2$. What are the decimals for $\frac{2}{5}$, $\frac{3}{5}$, and $\frac{4}{5}$. Why?

PLACE VALUE CHART**Practice**

Change the following decimals to fractions. Make sure to simplify each fraction. If you memorized the “Common Fractions/Decimals,” you do not need to show work.

1. .08

6. .00016 (see chart)

2. .004

7. -6.008

3. -5.025

8. 26.75

4. .375

9. -9.248

5. -9.875

10. .00065

Change the following fractions to decimals. Remember to reduce the fraction first. If you memorized the "Common Fractions/Decimals," then you do not need to show work.

11. $\frac{7}{100}$

17. $-10\frac{1}{3}$

12. $\frac{3}{24}$

18. $-9\frac{27}{72}$

13. $\frac{2}{5}$

19. $\frac{125}{100}$

14. $\frac{5}{8}$

20. $\frac{18}{24}$

15. $-8\frac{3}{10,000}$

21. $-\frac{5}{32}$

16. $\frac{3}{16}$

22. $\frac{22}{7}$

23. a. Change $\frac{1}{9}$ to a decimal.

b. Change $\frac{2}{9}$ to a decimal.

Finish the pattern.

c. $\frac{3}{9} = \underline{\hspace{1cm}}$, $\frac{4}{9} = \underline{\hspace{1cm}}$, $\frac{5}{9} = \underline{\hspace{1cm}}$, $\frac{6}{9} = \underline{\hspace{1cm}}$, $\frac{7}{9} = \underline{\hspace{1cm}}$, $\frac{8}{9} = \underline{\hspace{1cm}}$

d. So $\frac{9}{9} = \underline{\hspace{1cm}}$. But isn't $\frac{9}{9}$ the same as 1 whole?

Explain your thinking. _____

How to Tell if a Fraction Terminates or Repeats

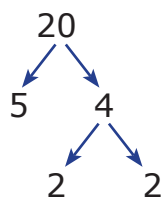
When changing a fraction to a decimal, it helps to know ahead of time if the decimal will terminate or repeat. To figure this out, follow these steps.

Example 1: Does the fraction $\frac{6}{40}$ terminate or repeat when changed to decimal?

Step 1: Reduce the fraction first! It becomes $\frac{3}{20}$.

Step 2: Do a prime factorization of the denominator. Do not worry about the numerator.

The prime factorization of 20 is: $2 \cdot 2 \cdot 5$ or $2^2 \cdot 5$.



Step 3: If you have only factors of 2, or only factors of 5, or both, but no other primes such as 3, 7, 11, etc. in the prime factorization of the denominator, then the fraction will terminate when changed to a decimal.

Example 2: Does the fraction $\frac{3}{70}$ terminate or repeat when changed to a decimal?

The fraction is already reduced. The prime factorization of 70 is $2 \cdot 5 \cdot 7$. Therefore the prime number 7 will make this fraction repeat when changed to a decimal repeat.

Practice

- Why do you think having only the primes 2 and/or 5 and no other primes will make the fraction terminate when changed to a decimal? Explain your thinking.

- Do these fractions terminate (T) or repeat (R) when changed to a decimal? Use an extra sheet of paper if needed. Don't forget to reduce each fraction first.

a. $\frac{1}{20} =$ _____

b. $\frac{3}{30} =$ _____

c. $\frac{22}{14} =$ _____

d. $\frac{1}{64} =$ _____

e. $\frac{9}{54} =$ _____

f. $\frac{8}{41} =$ _____

Ratio, Rate, and Unit Rate

A **ratio** is the relationship between two quantities.

Example 1: In a classroom there are 15 girls to 7 boys. What is the ratio of girls to boys?

You can write the answer as 15 to 7, 15:7, or $\frac{15}{7}$.

Since the question reads “girls” to “boys,” then the number that matches with girls goes first. If you write the ratio as a fraction, you can reduce.

Example 2: In the problem above, what is the ratio of boys to the total number of students in the classroom?

The answer is 7 to 22, 7:22, or $\frac{7}{22}$.

Example 3: At a dance the principal of a school wants to have the ratio of chaperones to students to be 2 chaperones for every 9 students. If the total number of chaperones plus students was 110, how many chaperones were there?

2 chaperones + 9 students = 11 people. $\frac{2}{11}$ is the ratio of chaperones to total people.

So $\frac{2}{11}$ of 110 must be chaperones. $\frac{2}{11} \cdot 110 = 20$. There are 20 chaperones.

Example 4: Dana drove 200 miles using 5 gallons of gas. Simplify the ratio of miles to gallons.

$\frac{200 \text{ miles}}{5 \text{ gallons}} = \frac{40 \text{ miles}}{1 \text{ gallon}}$ You can say Dana’s car gets 40 miles per gallon or 40 mi/gal or 40 mpg (miles per gallon).

$\frac{200}{5}$ is called a rate (a special type of ratio used to compare measurements with different units) and $\frac{40}{1}$ is called a unit rate.




A unit rate is a rate with a denominator of 1.

Practice

Find the answer to these problems using these bathroom tiles. Be sure to reduce if possible. Use a separate sheet of paper if needed.



1. What is the ratio of yellow shapes to squares? _____
2. What is the ratio of red triangles to total triangles? _____
3. What is the ratio of green triangles to green squares? _____
4. What is the ratio of triangles to squares? _____
5. What is the ratio of triangles to total shapes? _____
6. If the design above has to be repeated multiple times on a bathroom wall to make a border and the final design has 104 total shapes (squares plus triangles), how many triangles are there on the banner? Show your work.
7. Dan drove his car for 250 miles and used 10 gallons of gas. Elsie drove her truck for 300 miles and used 20 gallons of gas. DJ drove his motorcycle 120 miles and used up 3 gallons of gas. You can also use the abbreviation "mpg" for miles per gallon. Complete the table below.

Vehicle	Rate	Unit Rate
		
		
		

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8. If gas costs \$2.30 per gallon, how much did each trip in problem 7 cost? Show your work.

a. Dan’s trip _____

b. Elsie’s trip _____

c. DJ’s trip _____

9. A banana bread recipe calls for 1 cup of sugar for every 2 cups of flour. Finish the following table.



Flour (Cups)	Sugar(Cups)	Flour and Sugar (Cups)
2	1	3
3		
6		
		12
	5	
		$16\frac{1}{2}$

10. The same recipe above calls for $\frac{1}{4}$ tsp of salt for every 2 cups of flour. If Mr. Baker used $1\frac{1}{2}$ tsp salt, how many cups of flour should he use? Show your work.

11. Find each unit rate.

a. The average human walks 9 miles in 3 hours.

b. Michael Phelps swims 3 miles in a $\frac{1}{2}$ hour.

c. Katie Ledecky swims 800 meters in about 8 minutes.

d. A gray wolf runs 70 miles in 2 hours.



e. Usain Bolt runs 55 miles in 2 hours.

f. A cheetah runs 245 miles in $3\frac{1}{2}$ hours.



12. Fill in the table below.

Two 8" pizza pies for \$21.90.



Number of Pizza Pies	Cost
1	
2	\$21.90
3	
16	

13. In problem 12, the dependent variable is the "cost" and the independent variable is the "number of pizzas." What do you think is meant by those terms? Explain your thinking. Hint: Think of independent as free to choose what you want. A variable is something that changes.
