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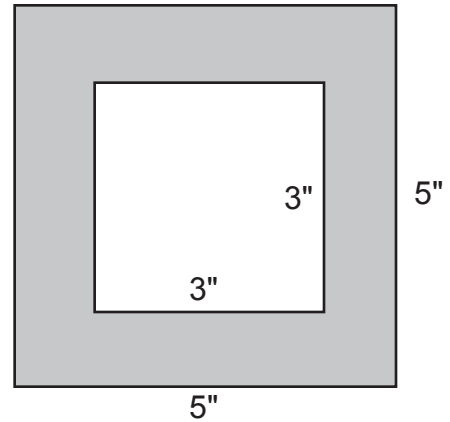
# 55. Percent Covering



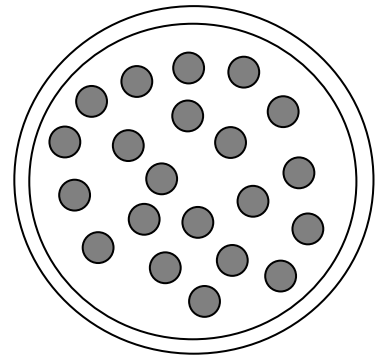
Name: \_\_\_\_\_

If needed, round final answers to nearest percent.

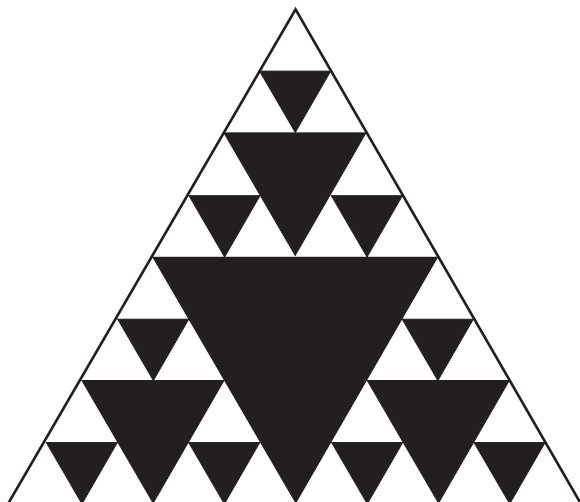
1. What percentage of the 5" x 5" square is shaded?



2. A large circular pizza has a 16-inch diameter. It is covered by circular pepperoni, each with a 2-inch diameter, as shown. None of the pepperoni overlap. What percentage of the pizza's area is covered by pepperoni?

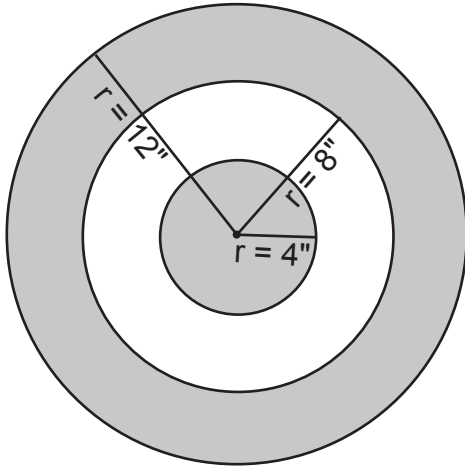


3. What percentage of the equilateral triangle is shaded? Figure is drawn to scale.

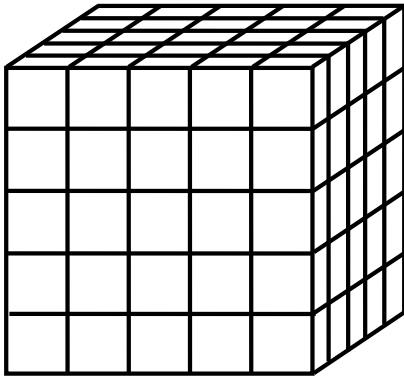


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4. What percentage of the 12-inch radius circle is shaded?

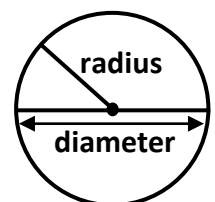


5. The 5 x 5 x 5 block of unit cubes is submerged in yellow paint. What percentage of the unit cubes has at least some yellow paint on them?



**Math Facts**

1. A circle's area equals  $\pi$  times the square of the radius, where  $\pi = 3.14159\dots$
2. The lengths of the sides of an equilateral triangle are equal.

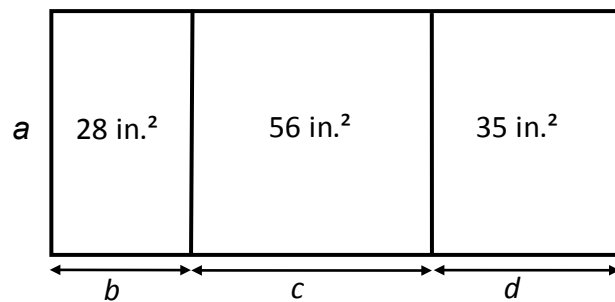




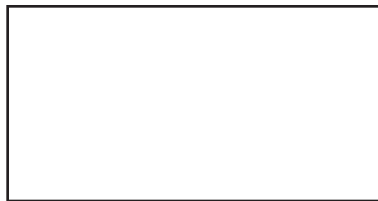
### 74. Dimension Detective

Name: \_\_\_\_\_

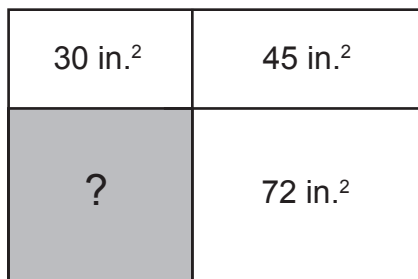
1. The rectangle areas are given. The lengths  $a$ ,  $b$ ,  $c$ , and  $d$  are whole numbers, and  $a$  is greater than 1. Determine the values of  $a$ ,  $b$ ,  $c$ , and  $d$ .



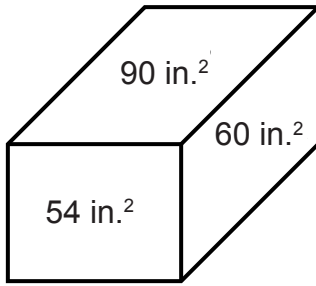
2. For what whole number values of length and width will the rectangle have an area of 60 square yards and a perimeter of 38 yards?



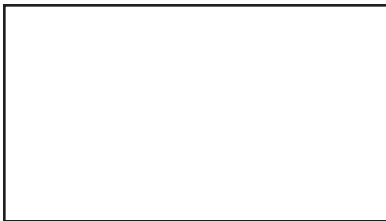
3. The area of each unshaded rectangle is given. What is the area of the shaded rectangle?



4. The areas of the rectangular faces of the box are shown. What is the volume of the box?



5. A rectangle has an area of 48 square yards. For what whole number values of length and width will the rectangle have the smallest perimeter?



### Math Facts

1. The set of whole numbers is  $\{0, 1, 2, 3, 4, 5, \dots\}$ .
2. The area of a rectangle of length  $L$  and width  $W$  is  $L \cdot W$ .
3. The perimeter of a rectangle of length  $L$  and width  $W$  is  $2 \cdot L + 2 \cdot W$ .
4. The volume of a box of length  $L$ , width  $W$ , and height  $H$  is  $L \cdot W \cdot H$ .