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## THE TRICK: Who's Got What?

1. Select three people as subjects. Call them  $P$ ,  $Q$ , and  $R$ .
2. Place three items on a table. Call them  $a$ ,  $b$ , and  $c$ .
3. While your back is turned, ask each person to select one of the items and conceal it.
4. Place 24 counters (you can use poker chips) on the table. Give one chip to  $Q$  and two chips to  $R$ . Person  $P$  gets none.
5. With your back turned, ask whoever selected item  $a$  to take as many chips as s/he now has, whoever took  $b$  to take three times as many chips as s/he now has, and whoever has  $c$  to take nine times as many chips as s/he now has.
6. Face the subjects and ask one of them to carry out the following divisions with a number you suggest:
  - Divide the suggested number by 3, noting the quotient and remainder.
  - Divide the quotient by 3, noting the new quotient and remainder.
  - Divide the new quotient by 3, noting the quotient and remainder.
7. Now you can announce who is concealing each of the items.

### UNRAVELING THE TRICK

1. How many ways can  $P$ ,  $Q$ , and  $R$  make selections from the items  $a$ ,  $b$ , and  $c$ ?
2. Choose one of the possible selections found above and note the number of counters given to the subjects (3), the number taken by the subjects (call this number  $N$ ), and the number left on the table (call this  $T$ ). How can you determine the value of  $N$  if you know the value of  $T$ ?
3. For the selection used above, carry out the three divisions (in step #6 of *THE TRICK*) using the number  $N$  as your first dividend. What are the three remainders?
4. Let the numbers 0, 1, and 2 be associated with the three subjects  $P$ ,  $Q$ , and  $R$ , respectively ( $P = 0$ ,  $Q = 1$ ,  $R = 2$ ). Compare your three remainders (found in #3 above) with the identities of the persons who selected items  $a$ ,  $b$ , and  $c$ . Can you predict who took each item?

## QUESTIONS TO PONDER

1. Does the number of counters used in the trick have to be 24?
2. What is the minimum number of counters needed in the trick?
3. How many remainders are possible when a given number is divided by 3?
4. After the three divisions are carried out, will any two remainders be equal?

## FURTHER INVESTIGATIONS

1. Suppose there were four subjects ( $P$ ,  $Q$ ,  $R$ , and  $S$ ) selecting from four items ( $a$ ,  $b$ ,  $c$ , and  $d$ ). Also, suppose you gave 1 counter to  $Q$ , 2 to  $R$ , 3 to  $S$ , and 0 to  $P$ . Determine how you would alter the trick so that you could predict which subject selected each item. (Hint: Have a subject take either the same number of counters, 4 times as many, 16 times as many, or 64 times as many as s/he has.)
2. Show how converting the number of counters taken by the subjects to a base four numeral could be used to predict who took what.
3. If there were 5 subjects, what is the minimum number of counters that would be needed? How could you alter the trick to solve the problem of needing large numbers of counters for your prediction?